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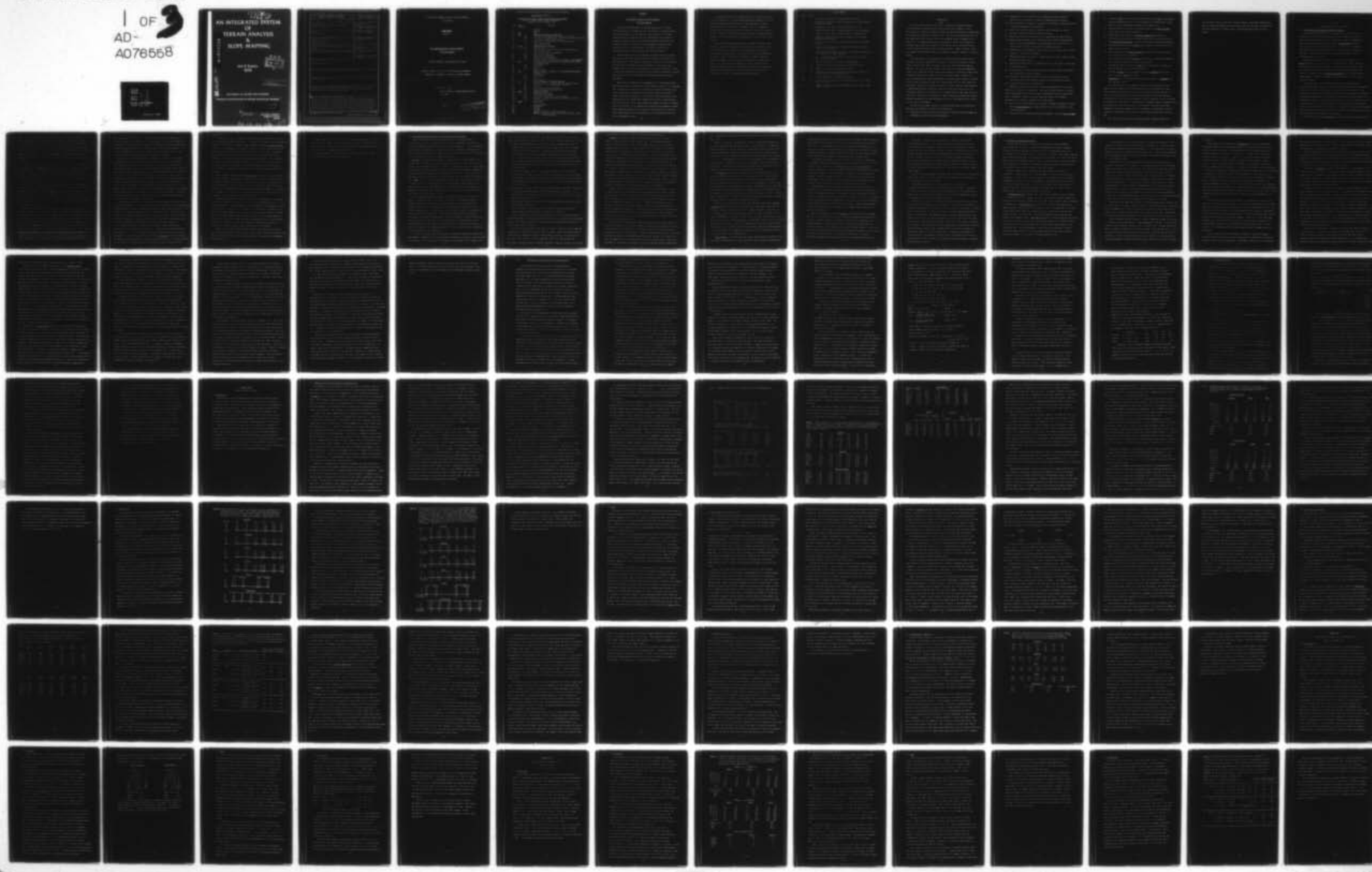

LEVEL II

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AN INTEGRATED SYSTEM OF TERRAIN ANALYSIS & SLOPE MAPPING

Ian S. Evans
1979

Final Report on DA-ERO-591-73-G0040

"Statistical characterization of altitude matrices by computer"

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DEPARTMENT of GEOGRAPHY
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A system is described which: (a) replaces existing sets of diverse terrain indices with a group of statistics for point-characteristics; (b) calculates all of these statistics in a single computer run from a single data set; and (c) utilizes available altitude matrix data. The procedures are applicable to altitude matrix data at any grid mesh. From altitudes in each 3 x 3 submatrix, a quadratic surface is fitted and solved for its first and second horizontal and vertical derivatives at the central point. This yields the

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'STATISTICAL CHARACTERIZATION OF ALTITUDE MATRICES

BY COMPUTER'

FINAL REPORT

(REPORT 6)

An integrated system of terrain analysis

and slope mapping

THE FINAL REPORT ON GRANT DA-ERO-591-73-G0040

8 Jun 73 - 4 Sep 79

By IAN S. EVANS, M.A., M.S., Ph.D.,(Principal Investigator)

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To: Dr. W. Grabau

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U.S. Army.

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1979

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Statistical characterization of altitude matrices by computer

FINAL REPORT (REPORT 6):

AN INTEGRATED SYSTEM OF TERRAIN ANALYSIS AND SLOPE MAPPING

by IAN S. EVANS, 1979, with the assistance of
MARGARET YOUNG and JASBIR S. GILL

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ABSTRACT

An Integrated System of Terrain Analysis and Slope Mapping

A system is described which (a) replaces existing sets of diverse terrain indices with an integrated group of statistics for process-relevant point characteristics; (b) calculates all these statistics in a single computer run from a single data set, thus achieving practical as well as conceptual simplicity; and (c) utilises altitude matrix data which is now becoming available as a by-product of photogrammetric automation, thus reducing the cost of data acquisition. The system is applicable to altitude matrix data at any horizontal resolution (grid mesh).

From altitudes in each 3 x 3 submatrix, a quadratic surface is fitted and solved for its first and second horizontal and vertical derivatives at the central point. This yields, in addition to altitude, the slope gradient and aspect, profile convexity and plan convexity at every point in the original matrix, except for the peripheral row and column on each side. Each of these five characteristics has an important influence on geomorphic processes and human activities, and is encountered in deductive modelling of slope development.

These 'point' characteristics are presented as (i) line-printer shaded maps, with the option of graph plotter contour maps in addition; (ii) histograms; (iii) scatter plots of each pair; (iv) matrix of pairwise correlations, plus circular regressions on aspect and several multiple regressions; and (v) summary (moment-based) statistics. (iii) and (iv) show up any unusual local characteristics, but confirm that in general the five basic characteristics have little relation to each other, except that gradient is usually a quadratic function of altitude. It is suggested that previously used geomorphic indices can be expressed in terms of one or several of the statistics (v), and that this routine analysis of an altitude matrix is of considerable practical as well as geomorphologic value.

A comparison is made with other approaches, such as spectral analysis and fractal modelling. The long-distance persistence properties of terrain mean that considerable extra variance at long wavelengths is usually incorporated when the study area is extended. Hence, for example, the autocorrelation function varies with the length of series or size of area studied. Variances of derivatives are also affected, but means, skews and kurtoses are not: derivatives are more sensitive to grid mesh than to size of area.

The statistics proposed are replicable in respect of different grid incidences, except that higher moments of convexity (especially in plan) are unstable because of long tails of extreme values. A grid mesh of 100m is useful for glaciated mountains, but 25m or finer is desirable for many areas. Areas of less than 5 x 5 km may be too small to provide replicable estimates of the land surface properties of a broader region. Results from eight matrices are presented. Three of these are from glaciated mountains, which are characterized by high mean and variability of gradient, high variability of altitude, positive skew of profile convexity and an excess of strong positive plan convexities.

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Chapter One

AIMS

This is the final report on a project to implement and evaluate techniques for the statistical characterization of altitude matrices by computer. Project objectives were to improve, integrate, automate and evaluate techniques for summarisation and portrayal of land surface characteristics. The choice of altitude matrices as the most appropriate form of data for input to an automated system is defended here in section (2c). A program package was to be developed to process altitude matrices and supply maps, graphs and statistics for the vertical and horizontal derivatives of the altitude surface and for their interrelationships. Graphic portrayal, mainly as histograms and area-shaded maps, was considered an important complement to numerical summary.

Land surface was to be characterized by quantitative descriptors applicable to terrains of any origin, regardless of the specific forms such as channels, taluses, dunes or drumlins which might be present; in this way, diverse terrains might be compared. The project thus fell entirely within the field of general geomorphometry (Evans, 1972), analysing the land surface as an example of a continuous, rough surface in terms of attributes at a sample of points or arbitrary areas. Compared with specific geomorphometry, general geomorphometry completely avoids problems of landform definition and delimitation (Evans, 1974: Evans and Cox, 1974), so that subjectivity and operator variance can be reduced drastically. The main problem is the dependence of general geomorphometric results upon data resolution, and this also was to be investigated together with the possibility of standardisation in grid mesh and size of study area. More explicit investigation of spatial scale by two-dimensional spectral analysis was proposed; this was considered in the first three Reports and is touched on only briefly in this Report.

This was a basic research project, but the resulting system for geomorphometric description and analysis is intended to be valuable for:

(i) comparison (ordination and/or classification) of different terrains, suggesting differences in form which require explanation;

(ii) background data for military and civil engineering studies concerned with trafficability, construction, irrigation, drainage and erosion hazard;

(iii) data for studies of climate, soil, vegetation and farming, including comparison of farms and assessment of land potential;

(iv) ground truth data for studies of terrain by remote sensing, e.g. radar;

(v) routing of terrain-following aircraft and missiles;

(vi) statistical studies of intervisibility in targeting and telecommunications;

(vii) demonstrating the extent to which sites selected for detailed geomorphologic study (e.g. process measurements) are representative of a study area or of a certain subset of slopes which comprise the target population;

(viii) demonstrating how representative a study area is of a broader region to which extension of conclusions is desired;

(ix) expressing the areal importance of a target population (e.g. 'straight slopes') selected for detailed study;

(x) predicting process rates and discharges from their relationships to attributes of land surface form;

(xi) providing data for input to empirical models simulating landform development, e.g. slope erosion or glacial erosion;

(xii) showing the presence and importance of various combinations of surface attributes, which may be relevant to predictions from theoretical models;

(xiii) monitoring the quality of altitude data.

Commitment to this project implied that previous approaches were not entirely satisfactory in relation to these goals. Their main problems were lack of clarity in defining separate attributes, clumsiness in data generation and processing, and lack of integration. Review of previous approaches (Evans, 1972) prompted consideration of a number of desiderata for the new system:

(a) each point on the land surface (or at least, its mapped representation) should have an equal probability of being considered, with no bias related to local surface properties;

(b) the distribution of data points should be systematic, to provide even coverage;

(c) the spatial scale at which properties are measured and the units of measurement should be recognized explicitly, without generation of 'dimensionless' ratios based on variable denominators (Mosimann, 1970; Chayes, 1971);

(d) calculations should be based on data sources likely to be widely available, without special data capture operations;

(e) properties should be capable of rapid and inexpensive determination;

(f) the properties defined should not simply be descriptive indices; they should have real functional meaning in terms of land surface processes and operations;

(g) underlying concepts should be simple and the properties defined should not be capable of sub-division;

(h) each property should be clearly separated from the others and each statistic should convey distinct information;

(i) it is therefore likely that multiple descriptors will be required, recognizing the multivariate complexity of the real land surface;

(j) concepts and statistics should be robust to the complexities of nature, e.g. the non-stationarity of the real land surface;

(k) the basic concepts should fit together to form an integrated system, not a list of unrelated ad hoc definitions;

(l) determination of the properties and statistics should also be integrated, for efficiency and clarity.

It is maintained that the 'integrated system of terrain analysis and slope mapping' defined, exemplified and evaluated here fulfils the essentials of these twelve desiderata and that most of the aims of the project have been achieved. It would of course be desirable to collect further evidence, and the eight matrices discussed in this report form a data base which, while voluminous, is slim in relation to the ambitions of the project : nevertheless, a considerable amount of exemplification and evaluation has been achieved and it is now possible to provide a preliminary assessment of the system together with a full statement of the operations involved. This permits consideration of the direction which further research might fruitfully take.

This project, then, has provided a fully automated, integrated system for

calculating the distinct properties of surface geometry in each small neighbourhood, and for summarising, mapping and interrelating these within an area, thus permitting effective comparison of different areas. There are many applications for such a system.

Chapter Two

THE BASIC PROPERTIES OF LAND FORM

(2a) Altitude and its derivatives, slope and convexity

In any scientific discipline, it is essential first to define the basic terms and then to observe relationships between the fundamental variables. No sophistication of methodology, no vast accumulation of data can compensate for poorly defined variables and obscure concepts. Geomorphologists attempt to understand the form of the land surface, mainly in relation to processes and materials: hence the terms in which that form is conceived are of fundamental importance. What attributes of form are of greatest relevance to surface processes and man's activities?

Here we are concerned with definitions of surface form which are of general relevance - in fluvial, glacial, cryonival, eolian, coastal, submarine and even lunar and martian environments. We need techniques to measure these attributes efficiently, and to express their inter-relationships simply and clearly. This study is termed 'general geomorphometry' while the term 'specific geomorphometry' is applied to analyses of landforms specific to one or more of these environments.

The surface which is to be analysed is defined by deviation from the equipotential surface of sea level. The latter is approximately an oblate spheroid, but since at any one time we are usually concerned with small parts of it, the equipotential surface will be treated as a flat plane on which position is defined by the cartesian coordinates (x,y). Deviations (z) are measured at right angles to the equipotential surface. A very desirable simplification is that the surface should be 'single-valued', with one and only one value of z for each position (x,y): thus we leave cave and tunnel forms, and overhangs, to be considered by specific geomorphometry. Despite the resulting vertical cliffs, the land surface will be treated as continuous. Unlike some theoretical surfaces, it is *one*-sided.

The value z is known as the altitude of the land surface, and obviously it must

be the first of our fundamental attributes. It controls air pressure and potential energy (with reference to the given base level, sea level). Altitude is a dominant influence on temperature and a major one on rainfall and snowfall. In process terms, the importance of altitude is undeniable, and yet in many ways spatial rate of change of altitude is even more important: scarcely a paper in geomorphology fails to make some reference to this concept. It is known as <u>gradient</u>, and the gradient at any point is measured in the direction in which rate of change of altitude is greatest, i.e. along a 'slope line'.

In any other direction - along an arbitrary line of traverse, for example - we are dealing only with <u>apparent</u> gradient, or one component of gradient. For example, Demirmen (1972) dealt with apparent gradient and curvature along a number of radial profiles. This produces an undesirable proliferation of surface descriptors with no individual importance, and should be avoided unless only profile data are available (as in some early lunar studies; Rowan, McCauley and Holm, 1971).

Gradient has been expressed on various measurement scales - often as a tangent*, a vertical distance divided by a horizontal one- but it is most naturally and neutrally expressed (Tricart, 1965, p.166) as an <u>angle</u>, the angle between the horizontal (i.e. the equipotential surface) and a plane tangent to the surface at the given point. It then varies between 0 and 90 degrees, whereas tangent (gradient) has no finite upper limit. Gradient controls the operation of gravity on bodies on the land surface; vehicles, stones, soil, water, snow or ice.

Sometimes the vaguer term 'slope' is used (e.g. Young, 1972, 1974) for what is defined here as gradient. This has much broader connotations (hillslope, valley slope) and in morphometry it is useful to define <u>slope</u> at a point as the properties of the plane surface tangent to the surface at that point. Gradient is one such property: but this slope is a vector and a second property is required to specify slope completely. This second property is <u>aspect</u>, the direction in which a frictionless ball starting from rest would move down the surface. Aspect is expressed either as a point of the compass <u>(northwest, south-southeast) or, more precisely, as degrees measured clockwise from</u>

* The term 'gradient' is often reserved for the tangent of angle, with 'slope angle' used for gradient: however, it seems unnecessary to have different words for the same variable expressed on scales which have simple monotone relations to each other.

north (southeast = 135°, west 270°): note that the latter scale of measurement is circular, its end (360°) is identical to its beginning (0°) since this dividing point is arbitrary. As a purely abstract property of a mathematically-defined tangent plane, aspect may appear unimportant - much less important than altitude and gradient - but on the oblate spheroid of the real world aspect shares with gradient control of the incidence of directional agents, above all wind and solar radiation.

Aspect cannot be viewed independently of gradient: if gradient is zero, aspect is indeterminate, and the importance of aspect increases (initially) with gradient. Their separation, however, is not arbitrary since definition of the horizontal is fundamental and deviation from it is relevant to geomorphic processes: gradient affects the operation of gravity and aspect does not, so the conceptual separation is clear-cut. Definition of slope in terms of gradient and aspect is analogous to the practice of defining structural surfaces in rocks in terms of the 'poles to the plane'; the aspect (outward orientation) and gradient (inclination) of this pole can be represented by a plot in polar coordinates, with zero gradient at the centre (Chapman, 1952). For a geomorphologist, definition of the plane of slope by gradient and aspect is more tangible and more attractive than an alternative such as the coefficients a and b of the plane surface equation $z = ax + by + c$, where a and b are the (tangent) apparent gradients in two directions at rightangles to each other.

Gradient and aspect are the two components of the first spatial derivative of the land surface, relating to the vertical and the horizontal respectively. Their interrelated importance naturally prompts the question: are further (higher-order) derivatives of the surface of any interest to geomorphologists? The answer so far is - yes, the second derivative of the surface has very often been invoked. Although Tricart (1947) defined an 'Index of symmetry of curvature' which reflects a third vertical derivative, there is as yet no geomorphologic interpretation for third and higher derivatives.

The second derivative of a surface is its curvature, commonly described by geomorphologists as 'convexity' and 'concavity'. (Fortunately all three words may be abbreviated to 'c'). It is convenient to measure convexity as positive and concavity as negative on the same scale, so the use of the term <u>convexity</u>, adopted here, conveys the meaning of the sign convention: concavity is negative convexity. This sign convention follows Young (1972) and many others, but is contrary to that of Troeh (1965) and

Speight (1968).

Convexity is the rate of change of slope, but we have seen that slope is a vector with two components, gradient and aspect. It is useful to recognize the two corresponding components of convexity. The rate of change of gradient is known as profile convexity ('profc'): this accelerates (or decelerates) fluxes of water and sediment, and affects the stress distribution both in the rock and in any snow or ice cover (convexities are more liable to failure). Convexity and concavity of slopes in profile figure extensively in discussions of slope development (Carson and Kirkby, 1972). The concept of 'relative curvature' as defined by Ahnert (1970) is not helpful because it masks the dimensionality of an essentially scale-dependent phenomenon, as does Demirmen's (1975) definition of curvature of a slope profile. Neither of these were tested with real data.

Rate of change of aspect is known as plan convexity ('planc') and can be thought of as contour curvature. Although many field and theoretical studies of slopes have, for convenience, excluded this factor and considered slope profiles, or confined attention to slopes 'straight in plan', a number of authors such as Sparks (1960, p.67-68) have insisted on the importance of the third dimension controlling divergence (in convexities) and convergence (in concavities) of water flow and sediment flux. Plan convexity, then, is essential to a full expression of land surface form.

Although both components of convexity can be variously expressed, it is convenient to retain the convention of Young (1964, 1972, 1974) who used degrees per 100 m for both: this gives numbers of convenient magnitude. For some purposes (e.g. topographic shelter) it may be useful to combine the two components. Since they are in the same units (and the maximum feasible change at a point is 180° in either gradient or aspect), planc and profc could be added together; planc, however, tends to be of much greater magnitude. Preferably, altitude at a point may be related to average altitude at a fixed distance (Evans, 1972), However, it is more often necessary to separate plan and profile components of convexity because of their different effects on processes.

The basic set of surface properties thus defined are interrelated in derivation, yet convey distinct information about a point on the surface. The properties are altitude itself, and four derivatives of the surface so defined. The origins of this system are traced by Evans (1972).

Aandahl (1948) and Curtis et al. (1965) recognized the importance of all five variables for soil studies; Speight (1968) and Ahnert (1970) used slope and both types of curvature. A set of very similar definitions was independently provided by Krcho (1973) with his application of field theory.

Figure 1 summarises the terms defined so far, their relationship to each other, their units and their theoretical ranges.

(2b) Importance in geomorphology, in military science and in economy

These five properties having been defined as basic to any understanding of land surface form, fuller illustration of their practical importance in three fields will now be attempted: geomorphology, military science, and economy. Since the mechanisms involved overlap considerably, the three fields will be considered simultaneously; the five properties are discussed in the same order as in the previous section. This account is a brief summary, and far from a comprehensive review.

Altitude is important mainly because of its major effect on climate (Baker, 1944): although rates of change with altitude (lapse rates) do vary, the control of altitude is so strong that measurements are often corrected to a 'standard altitude' such as sea level. Thus altitude is often discussed as the major variable controlling pressure, temperature, wind and precipitation (Smallshaw, 1953; Steinhauser, 1967; W.M.O., 1973; Flohn, 1974). The latter three variables have very important geomorphologic implications (Tricart et al., 1962; Shcheglova, 1974). The secondary effects on hydrology of snow (Meiman 1970) and ice (Østrem, 1974, 1977) cannot be ignored in any discussion of spatial variations of glacier balance (Schytt, 1967; Dugdale, 1972). The effect of altitude on vegetation is emphasised by Daubenmire (1954), Bormann et al. (1970), Troll (1972) and Wardle (1974), and that on soils by Costin et al. (1952).

In a different way, relative altitude above a base level controls the potential energy of running water and of rock masses. Although for rock masses this energy is largely theoretical, the probability of being dislodged in the very long-term does increase with altitude. The 'base level' for most land areas is sea level, but for inland drainage basins it is the altitude of their lowest point and this fluctuates with deposition and with eolian erosion. Since both types of base level are changed by continuing earth movements, and sea level is further influenced by climatic factors, the long term influence of base level is less important than its immediate effect on the energy expended by running water as it moves to base level. A numerical simulation by Leopold and Langbein (1962) suggested the effect of vertical (and horizontal) constraints on the development of river long profiles.

The effect of altitude on man has long been of interest to geographers (Peattie,1936). Measurements by Staszewski (1957) show the great concentration of mankind at lower altitudes, even allowing for the concentration of land area at such altitudes. 56% of mankind live

below 200 m and only 1.5% above 2000m. Although some 10 million live above 3000 m, these are almost entirely within the Tropics. At least five causal factors are involved -

(i) Higher altitudes are unhealthy for man; above 5200 m, steady deterioration is suffered due to low partial pressure of inspired oxygen (Grover 1974). Between 3000 and 4000 m, most humans can acclimatise unless the ascent from sea level is too sudden : it is advisable to spend a few days around 2800 m, on the way up. Daily work output is reduced, however, except for those born and raised at such altitudes; they develop an athletic build. Since adverse effects are slight for normal humans below 3000 m, this cannot explain man's concentration at much lower altitudes.

(ii) In middle and higher latitudes, climate generally deteriorates with altitude so that a narrower range of crops can be grown or animals kept. This agricultural effect seems to be the dominant one, since in the Tropics man is often concentrated at intermediate altitudes, which provide favourable conditions with few pests than at lower altitudes.

(iii) Earth surface processes, mainly slope and fluvial processes, transport loose material from high altitudes and deposit it lower, ultimately near 'base level'. Hence high altitude soils are often rocky, while better soils are found at low altitudes, again favouring agriculture.

(iv) Many areas were settled from the sea, or by maritime-oriented peoples, who settled coastal areas more densely.

(v) Many of the great industrial cities of the last few centuries have grown as seaports, generating large coastal concentrations of population.

From the combination of these five distinct effects, it is not surprising that the relationship between altitude and population distribution is a complex one, varying between regions. Nevertheless, altitude is an important factor, operating mainly indirectly via climate, agriculture or position.

Altitude has more important direct effects on certain of man's activities, especially transport, and is therefore of great military importance. Reduced air pressure reduces the efficiency of engines and therefore maximum speed is reduced. The performance of artillery is also affected. Mountain ranges are barriers for light aircraft and helicopters, and passes assume considerable importance. Moreover, the effect of altitude

on climate makes different clothing and different rations appropriate.

Gradient is important throughout geomorphology and almost all continuous models of slope development include a gradient term. The downslope component of gravity is proportional to the sine of the gradient, and this is a factor in mass movement, either surficially or by deep-seated failure. The same factor produces a downslope force on running water; with frictional turbulence, this controls water velocity, which is the main control on erosive power. Frictional resistance, on the other hand, increases with the cosine of gradient. Because sine and cosine sometimes combine, e.g. in assessment of the safety factor for surficial mass failure, the tangent of gradient enters stability equations. It also enters directly for rates of 'lift and drop' processes, e.g. frost creep (Takeshita, 1963) and finally, it is often used because for the modelling of slopes below 20° in gradient, tangent is a reasonable approximation to sine. Runoff erosion is proportional to a power of gradient between 1.35 and 2.00, although a quadratic relationship is sometimes used (Hudson, 1971 p.184, D' Souza and Morgan, 1976) and further work is required to specify this relationship properly; for example, zero loss should be predicted for zero gradient.

Gradient is a major influence on agriculture. Each type of machinery, or type of draught animal used in ploughing, has a limiting angle for safe working. A table was provided by W.J. West for Curtis et al.(1965, p.28) showing varied limits, with some clustering at 17°, but few thresholds are abrupt: see also Macgregor (1957) and Crofts (in Cooke and Doornkamp, 1974, p.361). Gradient has an especially subtle influence on the design of irrigation systems. There is also an indirect effect via natural drainage: waterlogged soils on low gradients (especially at low altitudes) may be less tractable than well-drained soils on moderate gradients. High gradients, however, apart from their other difficulties, encourage over-rapid drainage and loss of soil, further deterring agriculture.

The simplicity of building on flat land, coupled with the increasing preference of industry and retailing for horizontally extensive single-storey layout,makes flat land a valuable resource for construction. This use can usually outbid agriculture, with its preference often for the same land. Conversely, steep hillsides are used for building when there is pressure from an expanding urban area (e.g. Oakland) or for amenity (e.g. around Los Angeles), but a considerable extra cost has to be paid either on extra site investigation, foundations and drainage, or by later catastrophic events (Leighton,1966.

1976). Gradient was the only land form variable considered in a review of planning by Kiefer (1967).

The effect of gradient on construction of lines of communication reached its peak in the canal age, when contours were often followed slavishly. It remained an important constraint on railway construction, high grades limiting train speed and increasing running costs. Roads for routine motor vehicles have a maximum gradient of some 1 in 3 (18.4°): in Britain, Ordnance Survey maps distinguish hills exceeding 1 in 5 (11.3°: two arrowheads) or 1 in 7 (8.1°: one arrowhead), which are hazards for ailing vehicles. Considerable cut and fill is performed to reduce the gradients of modern roads. Off-road vehicles also have limiting gradients, although it will be argued below that surface curvatures may be even more important.

Slope aspect affects surface processes mainly via mesoclimate (Evans 1977b). This often influences the distribution of glacial and nival action (Barry and van Wie, 1974) and of soil moisture and runoff. Landsliding is often concentrated on the moister slopes (Beaty, 1956; Curtis, 1971), while the asymmetry of valley slopes is a complex and still controversial field (Melton, 1960; Kennedy, 1976). Mesoclimatic and hydrologic contrasts between aspects are often apparent in soil (Stepanov 1967) and vegetation, and they affect agriculture distribution and productivity in comparable fashion (Peattie, 1936). Effects on military activity are mainly indirect, for example, different moisture conditions may influence trafficability.

The effect of curvature has less often been considered: nevertheless, profile convexity appears in many models for the development of erosional slopes over time. If material transport is proportional only to tangent (gradient), lowering will be proportional to profile convexity expressed as rate of change of gradient. Profile convexity produces acceleration of movement. Arnett (1971) found that, near Brisbane, convexity had an effect similar to increased gradient, in reducing soil depth, B horizon thickness, and A horizon clay content. Brown (1937) suggested that profile convexity increased runoff erosion, as did gradient, slope length, relative height and southeast aspects. D'Souza and Morgan (1976) suggested that omission of profile convexity accounted for much of the error in soil loss predictions.

Plan convexity, on the other hand, has usually been excluded from models of slope development on the grounds that its inclusion would cause intractable complications.

Slope analysts often define their target population as 'straight' slopes, though the range of small plan convexities to be included is not stated except by Young (1974: $\pm$ 30°/100 m). Since straight slopes form a minor part of many landscapes, this restriction is most unfortunate and reduces the relevance of such studies to a level comparable to studies of 'perfect competition' by economists: such simplifications may form essential starting points, but we must proceed to greater realism.

The qualitative importance of plan convexity in controlling the convergence or divergence of surface flow was quantified by Kirkby and Chorley (1967). In 1972 Young could still write (p.177) "no measurements of the relation between plan curvature and the rates of surface processes are available". Some were provided by Mosley (1974). Moore and Thornes (1976) included plan convexity ('contour convergence -divergence') in their modelling of topographic controls on transport capacity. An important result derived by Smith and Bretherton (1972) was that on plan convexities, minor perturbations would be unstable, while on concavities they might develop further into gullies.

Since man tends to travel horizontally rather than along slope lines, his construction of roads tends to be affected by curvature in plan more than by that in profile. The greater the frequency of extremes of positive and negative curvature (the more crenulated the contours), the greater the expense of cut and fill to produce a road of acceptable sinuosity and gradient. Plan convexity is also the component of surface roughness which most hinders overland movement: a steep but steady gradient is less troublesome than a rapid alternation of reversed slopes, and it may be passable by traversing.

There are some respects in which curvatures in plan and profile work together. Both influence the stress field of rocks: convexity increases stress release while concavity increases containment. The result is that profile convexities such as mountain shoulders or summits are more likely to fail, producing subsidence and sometimes catastrophic rock avalanches (e.g. Turtle Mountain, Alberta). A model study of slope failure by Stacey (1975) showed that increasingly sharp plan concavity reduces the volume of material involved. Of course, the concentration of moisture in concavities encourages failure and this may override the purely geometric effect.

Both types of concavity may provide shelter or shade from wind and rain (Lee and Baumgartner 1966): this is especially important in glacier balance and in the distribution of snowpatches (Young, 1976). The last remnants of ice, and probably also the first glaciers to form at the onset of a glaciation, are in sites of favourable aspect and altitude which are extremely concave in both plan and profile. Likewise, combination of the two concavities encourages the formation of frost hollows (Geiger, 1965). Shelter from wind, greater persistence of snow, and increased frost incidence have varied effects on vegetation, on agriculture and on heating requirements: hence the relation of these latter to concavities varies between regions.

Concavity is also an extremely important control on intervisibility, and hence on radar shelter. Here, as in other shelter effects, concavity at various scales in the surroundings of a point is involved.

The preceding summary of land form geometry effects is included to illustrate the practical importance of altitude and its derivatives. Calculation of their values provides more than simply a characterization of the landsurface in abstract terms: it gives a set of numbers each of which has a tangible reality and its own significance to surface processes and to man.

Nevertheless, it cannot be claimed that these values of any point on a surface are the only surface properties of such relevance. I have noted the importance of broader scales, especially in relation to concavity; specific calculations of 'shelter' at a point are required for specific purposes such as predicting glacier balance or degree of radar shelter. The length and height of a slope, measured along a slope line from divide to valley, have a great effect on surface runoff and on the stress field within a slope, and cannot be ignored in the study of soil erosion, landslides and avalanches. They are excluded from the present approach because their calculation involves threading routes through the neighbourhood of each point, sometimes to a considerable distance; this is a different type of computing to that needed for local derivatives. The relevance of slope profiles, though broad, is not quite so universal as that of altitude and its derivatives; in eolian landscapes, for example, their importance is reduced.

(2c) Data sources: sampling and accuracy

The efficiency and value of a terrain analysis system is largely dependent upon the data which it can use. Technical developments are continuously changing the opportunities available and the relative merits of different data sources, so it is unwise to form a dogmatic notion of their value. In Figure 2 the current three main data sources are recognised. Even though air photos are taken 'in the field' and require ground control, and topographic maps are based in varying proportion on air photos and on field observation, it is useful to distinguish these three sources of data since they have different implications for the next stage of processing.

Terrain information can be extracted from these sources either manually or automatically. Manual methods involve the measurement of profiles or point values, or the mapping of 'facets': any of these procedures can be performed by use of surveying instruments in the field, by use of photogrammetric plotting devices with air photos, or by use of protractor and ruler or dividers on topographic maps. Alternatively, a digital ground model (DGM) may be prepared for computer input, so that all further operations and calculations can be performed automatically.

A surface-specific DGM (Hormann, 1968 : Mark, 1975b) consists of (x,y,z) triplets of coordinates for points on changes in slope, on crests and on thalwegs, together with information identifying their neighbours so that they can be related to form (for example) a triangular mesh. Altitude matrices are DGMs in which x and y coordinates assume regularly spaced values defining a grid; hence only the z coordinates need to be stored along with grid (matrix) mesh and dimensions, given a convention for ordering the z values (e.g. start in the northwest corner and scan eastward along successive rows, ending in the southeast corner). Digitised contours are an even more special type of DGM, in which z is fixed while x and y vary: the resulting strings of (x,y) couplets define points along a contour, and are prefixed by a string delimiter and a header, z, for contour altitude. Like the three manual forms of data, the three types of DGM can in theory be derived from any of the data sources and can be converted to either of the other two types by various forms of interpolation (Rhind, 1975).

A surface-specific DGM can be established in the field by an extension of facet mapping, with relative (x,y,z) values recorded for computer input. An altitude matrix can likewise be generated from multiple transects, or (for microrelief) from a grid frame with vertical pins lowered on to the ground surface. Contours might be generated as in the nineteenth century by the laborious method of horizontal levelling, but with (x,y) coordinates recorded.

The problem with all these field-based DGM techniques is that data must be recorded for later entry into a computer. To this extent they are no more automated than the three approaches labelled 'manual'. DGMs may be produced more efficiently by computer input directly from machinery operating on air photos or maps. From air photos, this is done by automatic read-out onto tape (or cards): point, profiling and contouring modes lead to production of surface-specific, altitude-matrix or digital contour DGMs respectively. If maps are used, digital contours are produced using a digitising table for line-following; a surface-specific DGM is produced by encoding (x,y,z) triplets, possibly using a digitising table in point mode for (x,y) and typing in (z); and a matrix can be produced by software following raster scanning of contours, as by the Defence Mapping Agency TOPOCOM (Mays,1970), or by manual interpolation on a grid, with z recorded on tracing paper and then typed into a computer.

The point has been made, then, that any of the three manual types of analysis, and any of the types of DGM, <u>can</u> be derived from any of the three data sources. Some routes, such as the generation of digital contours from field surveys, are not widely used at present, but could easily become important with changes in technology. Because there are so many alternatives, assessment of their relative merits is far from trivial.

None of the manual methods (Young, 1972) can be recommended as means of providing full information on surface geometry. <u>Point sampling</u> means that each of the five basic properties must be measured separately, which is rather clumsy. Although gradient and aspect can readily be measured from air photos (Turner, 1977;Verstappen, 1977, p.57-62), accurate procedures are time-consuming and curvatures (convexities) are more difficult to measure. In fact, whatever the data source, point sampling of curvature probably requires measurement of gradient and contour trend on either side, followed by further

calculations.

Terrain analysis by the surveying of slope profiles in the field has received considerable attention from European geomorphologists in recent years; Young's (1974) review is the most up-to-date, with thorough definition of terms and discussion of practical problems, but see also Pitty (1969, 1970). (Neither author is entirely reliable on statistical analysis). Young (1974) discussed the problem of locating profiles for survey; if the profiles are to represent the whole region, or some portion of it such as 'straight slopes', they cannot be 'purposively' located; some random or systematic basis is essential. Surveying profiles upward from points randomly or systematically spaced along thalwegs (valleys) produces over-sampling of plan convexities (increasingly with height up the slope) and under-sampling of plan concavities. If, on the other hand, one surveys down from points on divides, plan convexities are under-sampled (increasingly down the slope) and plan concavities are over-sampled. The compromise suggested by Young is to start from points along a mid-slope 'slope sampling baseline': this reduces the magnitude of the sampling biases but by no means removes them. Mid-slope starting points may, however, be more difficult to locate in the field. Similar biases arise if slope profiles are surveyed through points located systematically or randomly.

A further problem may arise at both termini of a slope profile. The profile by definition follows a line of true slope: but if crest or footslope have relatively gentle gradients, the slope line curves to approach the divide or thalweg asymptotically. Some termination point is required, or divide and thalweg would be drastically over-sampled. Pitty (1969) terminated when profile gradient fell to equal the gradient along divide or thalweg; thus the latter were often left unsampled. Young (1974), on the other hand, continued the profile from this point to meet divide or thalweg at rightangles, thus mixing apparent slopes in with true slopes and producing a sudden break in profile direction, with a straight line section terminating the usually S-shaped plan of the main profile.

Slope profiling in the field has greatly increased our detailed knowledge of profile form and its relation to structure and process, and perhaps these problems and differences should not be overemphasised. They suggest, however, that it is inherently

impossible to produce a set of surface-specific lines (slope lines) which is an unbiased representation of an irregular surface. Though of great interest in themselves, slope profiles are poor in representing the surface of a region.

Identical arguments apply to profiles derived from air photos or from maps. These are usually more generalised, and straight lines from divide to stream are sometimes measured; this increases corruption by apparent slopes, and ignores the plan convexity. Information about plan convexity can be obtained by lateral sightings during field survey of profiles, but this is an unwelcome additional procedure.

The mapping of slope facets (Waters, 1958; Savigear, 1965) is appealing in that the whole land surface is covered. Less appealing is the almost untested assumption that change in slope is concentrated along mappable lines: this atomistic approach (Cox, 1978) is suitable in some parts of the English Pennines, but not in rolling country. Even when changes of gradient are abrupt, changes of aspect along contours are often gradual, making lateral delimitation of facets difficult.

From a statistical point of view, it is essential to provide measures of within-facet variability in all five basic properties: in the absence of such measures, claims that facet mapping provides 'complete' coverage of the land surface must be rejected. Facet área and shape necessarily vary, complicating analysis of relationships between properties. Where division into facets is well-marked, it should be demonstrable starting from a dense altitude matrix: on the contrary, it seems that morphological maps of facets contain a considerable subjective element, and are not repeatable (B.G.R.G. 1961, especially Appendices). Contemporary techniques to define breakpoints in slope profiles are totally unsatisfactory (Cox, 1978), and even improved techniques rarely divide slopes in the desired fashion. Maps of facets produced rapidly by Air Photo Interpretation may, nevertheless, provide useful sampling frames for soil and surface properties (Beckett and Webster, 1965).

Data from any of the 'manual' techniques can be typed into computer storage but, if the computer is to be used at all, input-output overheads make it efficient to enter the computer as early as possible and to perform as many data reduction operations as possible automatically (Roberts and Evans, 1970). Accordingly, and because of sampling or measurement problems with each of the 'manual' approaches, interest

here centres on computer approaches to terrain analysis, i.e. the use of DGMs.

Evans (1972, p.38-40) reviewed the morphometric use of digital contours; little progress has been made since. Contour divergence makes it difficult to identify lines of true slope; neither the method of Monmonier, Pfaltz and Rosenfeld (1966; see also Park, Lee and Scheps, 1971) nor that of Piper and Evans (1967) provides satisfactory measurements of gradient, and profile curvature would be still more difficult. Aspect and plan curvature can easily be calculated from contour trends, but contours oversample steep slopes, and frequency of occurrence must be corrected by dividing by tangent (gradient) before areas can be described. Grist and Stott (1977) compared four different types of DGM for a 1600 x100 m area in Wensleydale, Yorkshire, from the point of view of road engineering. Comparison was hindered because only densities of points and not costs of acquisition were given, but the results from altitude matrices were highly accurate for smooth or undulating ground. Those for a 15m mesh were much better than for 20m, but 10m and 5m gave diminishing returns. For rough terrain with sharp breaks, digital contours supplemented by break lines were considerably more accurate than altitude matrices, but included many more points. However, the use of bilinear interpolation for matrices by Grist & Stott necessarily smooths extrema: biquadratic interpolation should give more accurate results, and is reliable for altitude matrices (but not for irregularly distributed points).

The efficiency of surface-specific compared with matrix DGMs was considered by Mark (1975b) for manual digitisation of points from 1/50,000 maps of British Columbia. He compared 500m mesh 15 x 15 matrices with triangular meshes based on an average of 114 points, generated according to the system of Hormann (1968,1969). Mark found that though the triangular meshes took over five times as long to produce, the mean error of estimates of mean tangent (gradient) based on them was 2.8 times less, assuming that Wentworth's (1930) method gives the true mean tangent (it may overestimate). Mark's comparisons of error in estimates of relief and of hypsometric integral are less important, since both are based on extreme measurements (Evans, 1972). On the debatable assumption that mean error is in linear proportion to grid mesh, Mark estimated that for a given accuracy in estimating mean tangent (gradient), 2.7 times as much digitisation time is required for the grid method as for the surface-specific triangle method.

This conclusion is based on extrapolation, which is not supported even by Mark's own results: his Table 2 shows a mean error in relief for a 500m mesh grid only 0.289 (not 0.5) times that for a 1,000m mesh grid, while Table 5 gives 0.3 for mean errors in hypsometric integral: corresponding results for gradient are not given. Before Mark's conclusions can be considered seriously, they need support from results at the finer meshes (e.g. 180m, 40m) to which he extrapolates, and from different topographies.

Moreover, mean tangent (gradient) is not the only surface property of interest: what of altitude, aspect and convexity, and what of their variability as well as their mean? Though interesting, Mark's comparison of manual digitising times misses the point that matrices are becoming available as standard by-products of orthophotography, while there is no prospect of triangular mesh DGMs becoming available in this way.

Semi-automated orthophoto production is facilitated if the control altitudes are along profiles, rather than along break-lines, along contours or at random points. For this reason, a gridded DGM is being assembled for the whole of Sweden, with a mesh of 50m in the south and 100m in the north. (Ottoson, 1978). A grid is the most suitable form of DGM for a central data bank for different applications (Stefanovic, Radwan and Tempfli, 1977). Such data banks are increasingly important, and DGM data with its relatively long lifetime tends to form the backbone of geographic information systems. The measurement of a large number of equal-spaced data points (followed if necessary by data compression) is much less error-prone than the measurement of fewer points carefully (but subjectively) selected, especially in a production (as opposed to research) environment.

To date, there has been a preference for semi-automatic profiling in DGM and orthophoto production. In 1976, however, there was a breakthrough in the acceptance of automatic, analytical stereoplotters; six new models became available (Konecny, 1977). These can very easily generate gridded DGMs. For example, the Gestalt photomapping system iterates the correlation of left and right photographs 50 times per second until parallax is negligible (Kelly, McConnell and Mildenberger, 1977). For each patch, a 47 x 52 matrix of 2444 altitudes, spaced .182mm apart in the stereo model, is very quickly produced. Overlaps between patches are checked, and a stereo model can be completed in 90 minutes, yielding a 700,000-point DGM.

Using the Gestalt system, the U.S.G.S. has already produced 2,000 7½-minute single-model orthophotoquads at 1/24,000 (Southard, 1978). The amount and density of altitude matrix data promises, then, to be positively embarrassing. The question for the future is how to compact and process the great flood of gridded altitude data (Pfaltz, 1975; Jancaitis, 1978), rather than whether this is the best format in which to generate photogrammetric data.

Hence if programs can extract relevant terrain information with adequate accuracy from altitude matrices, the case for generating surface-specific DGMs specially is weak. Matrices also offer insight on the scale problem, which is crucial: surface-specific triangles necessarily vary in size and their results apply to a fuzzily-defined range instead of a single spatial scale. It will be demonstrated in chapter(3g)that surface properties are very sensitive to the spatial scale at which they are measured. Since any statement about such properties should be qualified by reference to the scale involved, approaches which cannot provide a precise answer to this question are at a severe disadvantage. This problem, together with considerations of data availability, makes it unlikely that very extensive comparative work on surface-specific DGMs will, or even should, be undertaken.

The advantages of grid sampling were recognised by Chapman (1952). Yates (1949) showed that a grid is the most efficient sampling scheme for an autocorrelated series (such as altitude). Workers on intervisibility, trafficability, radar, and terrain-following weapon systems have shown preference for gridded altitude data. Since funds available for geomorphometric research are more limited than for these related fields, it is sensible to take advantage of data availability by developing systems to process the digital data which they produce, namely, altitude matrices. Hence this report concentrates on the processing of altitude data for square grid meshes. The outline of the gridded area, however, can be either rectangular or irregular. Whether or not grids are the most efficient means of storing altitude data, they are the most easily manipulable data format for most high level computer languages. Also they minimise subjectivity and permit precise statements about the scale at which surface characteristics are measured and about the types of surface feature which will not be represented adequately.

The altitude matrix data used in this report have two origins. Data for the Cache area were produced automatically by UNAMACE, an early analytical stereoplotter, and processed by CONPLOT. There are special problems with this equipment, and the data have not been cleaned sufficiently; the effects of this are discussed in chapter 5 below.

The other data sets were produced manually by interpolation from contour maps, which had been produced photogrammetrically. They suffer from errors of interpolation, affected by the operator and the roughness of ground between contours, and errors of the original map, affected by photo scale, map scale, camera focal length and surface gradient (Richardus, 1973) and the reliability of the photogrammetrist and ground control.

Data accuracy is important because surface derivatives are based on local differences which are very sensitive to noise in the data, even when derivatives are calculated via a local quadratic. Data which represent altitude adequately may give inaccurate estimates of gradient and meaningless values for convexity. This is obviously important for substantive results, especially comparisons of different areas. Comparisons, especially of convexities, can be made only within a data type with error properties which are either constant, or vary monotonically in relation to real natural variations. Although the present report is methodological, it must be admitted that serious data errors do have implications for the techniques employed: for example, they might require data smoothing, or calculation of the local quadratic over a neighbourhood larger than 3 x 3. At present this does not seem necessary for the manually-interpolated matrices, but it probably is for the UNAMACE data.

The most accurate potential source of altitude matrices is from photogrammetric profiling, reading off point altitudes on a grid (preferably onto tape for direct computer input). This approach is obviously more direct than producing contours and then interpolating manually, and might reduce error by an order of magnitude; it is hoped to obtain some in the near future. The newer analytical stereoplotters may provide heights with an RMS error of the order of 1m. Meanwhile, it seems that some of the error produced by interpolation is not highly objectionable in that it smooths the surface, making a quadratic a better fit. Also, although it is difficult to obtain

relevant information, carefully-produced photogrammetric maps such as the Ordnance Survey 1/10,000 series may contain less error than the 'half contour interval' often quoted; in this respect they are much superior to the older maps discussed by Clayton (1953).

(2d) Calculation of derivatives from altitude matrices

It is asserted above that satisfactory (though generalised) estimates of surface derivatives (slope and convexity) can be calculated from altitude matrices. Obviously such estimates refer not to points, as we might ideally wish, but to local neighbourhoods covering several grid mesh units. This spatial generalisation means that the grid mesh must be sufficiently fine to define surface properties at the scale of interest, and that comparisons can be made only at a stated, constant grid mesh. It must be remembered, however, that in practice in the field surface derivatives are measured over finite distances: they are not point measures, and comparisons can be made only for the same measured length (Gerrard and Robinson, 1971: Gerrard, 1978). Calculation from altitude matrices usually (though not necessarily) implies generalisation over greater distances than measurement in the field, but the difference is quantitative rather than qualitative.

Techniques for calculating gradient from matrices have been proposed by Hobson (1967), Sharpnack and Akin (1969), Tobler (1969), and Struve(1977). The different technique used here and detailed in Report 5 is considered more advanced, in relation to two desiderata: (i) Gradient must be related to true slope lines, which may trend in any direction ; and (ii) slope and convexity should relate to the same local neighbourhood, centred on the point to which they are attributed.

Calculation of a derivative from a matrix involves generalisation over at least one grid mesh unit, since two or more neighbouring values must be considered. Minimum generalisation was achieved by Hobson (1967), who measured the slope over each isosceles triangle (in map projection) formed by a grid point (the apex) and its nearest axial neighbours in two adjacent directions. When all possible triangles of this type are considered, the plane is covered twice and there are four measurements per internal data point (plus two for each edge and one for each corner point in the square grid). If these measurements were to be attributed to points, they would be part-way along

the diagonals connecting data points: this is rather inconvenient and it would be feasible to calculate convexity for the same points.

A second possibility, covering a larger area but the same linear dimension, is to take squares formed by pairs of triangles and to average the two slopes or, preferably, to fit a least-squares plane to the four bounding data points. The fitting would simply involve calculating x and y components of apparent slope by comparing the altitudes of points pair by pair, then combining them by Pythagoras' Theorem. The resulting slope vector would be attributed to points in the middle of each square, equidistant from the data points. This is slghtly more appealing than the previous case, but again it seems inconvenient to calculate convexity for corresponding points. It is advisable, then, to calculate derivatives which can be attributed to the data points, thus facilitating comparison between first and second derivatives.

Struve (1977) compared three algorithms for slope (but not convexity) calculation, for cost and for accuracy in processing three mathematically-defined surfaces. The 'vector algorithm' simply compares apparent gradients to neighbouring points in eight cardinal directions, and takes the steepest; this is unacceptable since only eight values of aspect are possible, intermediate vectors being ignored (Struve's Table 1 incorrectly gives 2D as the x and y distance to diagonally related points, instead of D). The 'plane algorithm' compares four planes, each based on a triangle defined by altitudes at the central point and two of the nearest neighbouring points, (as in Hobson's scheme), and takes the steepest. In this case, any value of aspect is possible, but (as in the vector algorithm) gradient is measured to one side or other of the central point to which gradient is attributed, and not around it; thus condition (ii) above is not satisfied.

The third, 'three-dimensional surface' algorithm corrects this by comparing altitudes symmetrically disposed around the central point, which is itself ignored. Two independent estimates are made for apparent gradient in each axial direction, one based on the nearest neighbours (those along the axes) and another based on the two next nearest pairs (the diagonal

neighbours), and the maxima for each apparent gradient are combined (by Pythagoras) to give an estimate of true gradient. Although the effect of one apparent gradient being based on axial neighbours while the other comes from diagonal neighbours requires further investigation, this technique is much more acceptable. Not surprisingly, Struve (in his Table 2) shows this to produce reasonable precision for the analytical surfaces, while his other two algorithms produce totally unacceptable errors.

Having interpolated altitude at 45° intervals, Grender (1976) calculated slope from pairs of points one grid mesh from the point of interest and on opposite sides. He also looked at more generalised slopes, using a 30° interval for points 2 grid mesh units away, 15° for 4 and 5° for 10. Since each pair of points is symmetrical about the central point, the calculated value can justifiably be located at the central point. These are, however, apparent slopes, with a limited number of possible aspects, and even the maximum obtained is subject to the same deficiency as Struve's 'vector algorithm'.

If diagonal neighbours are excluded from Struve's 'three-dimensional surface algorithm', combination of apparent gradients from the two pairs of axial neighbours gives the 'finite difference' estimate of gradient defined by Tobler (1969). The latter is thus seen to be closely related to Struve's preferred estimate. By calculating gradient in the same way at each of the four axial neighbours, Tobler had extended this approach to provide an estimate of profile convexity (rate of change of gradient), which was not considered by Struve.

These 'finite difference' estimates were used in my study of the effect of grid mesh (Report 3), but were not considered ideal. Their deficiencies are (i) they do not make full use of the relevant information : heights at the central point and at diagonal neighbours are ignored in the calculation of gradient while heights at axial neighbours are ignored in the calculation of convexity; and (ii) while gradient is calculated over areas two grid mesh units wide, convexity is calculated over areas four wide and is thus more generalised. An extension of Struve's algorithm to calculate convexity would also extend the width of neighbourhood

to four grid mesh units. A different and slightly more sophisticated algorithm is chosen here, and is probably the optimal technique for calculating derivatives over a comparable neighbourhood which is as small as possible, so that the generalisation inherent in a grid-based approach is minimised.

To provide symmetry about the central data point, it is necessary to consider at least the nearest neighbours on either side, i.e. to cover an area two grid mesh units wide. The central point plus its four nearest neighbours adequately define slope, but provide only limited information on curvature. If altitude and the four surface derivatives are calculated from five altitude data points, five parameters are calculated from five data points : a curved surface plane is fitted to pass exactly through each data point, and no consideration is given to possible errors in the data.

If we go $\sqrt{2}$ further from the central point, we incorporate the four diagonal neighbours, making a total of nine data points forming a 3 x 3 submatrix. These are more than sufficient to fit the six parameters (a...f) of the full quadratic,

$$z = ax^2 + by^2 + cxy + dx + ey + f$$

The presence of three 'spare' data points means that the local quadratic surface is overdetermined, and does not need to pass exactly through the nine data points. This makes some (small) allowance for rounding and other errors in the data.

Although conceptually this surface is fitted by least squares, the procedure is greatly simplified by the arrangement of data on a square grid. The six parameters are calculated by multiplying the 9 x 1 vector of altitudes by a 6 x 9 matrix of coefficients: matrix inversion is unnecessary (Table 1, and Report 5, p.2). The derivatives of the quadratic surface at the central data point (0, 0 in the local cartesian coordinate system) are calculated by substitution in the quadratic equation; in fact, they can be expressed as simple combinations of the six parameters (Table 1). The formula for gradient turns out to be identical to that of Sharpnack & Akin (1969): it combines the apparent gradient estimates based on cornerpoints and axial points, which were considered separately by Struve (1977).For consistency,

Table 1 Definitions of derivatives in terms of square grid mesh g and the quadratic parameters a...f; and of the latter in terms of the altitudes $z_1 \ldots z_9$ (ordered from top left to bottom right: z_5 is altitude at the central point of the submatrix). Based on Report 5, p.3-4. Note that f and alt are in units of L; d,e, aspect and gradient are dimensionless; a,b,c are in units of 1/L; profc and planc in 1/100L.

$$a = (z_1 + z_3 + z_4 + z_6 + z_7 + z_9)/6g^2 - (z_2 + z_5 + z_8)/3g^2$$

$$b = (z_1 + z_2 + z_3 + z_7 + z_8 + z_9)/6g^2 - (z_4 + z_5 + z_6)/3g^2$$

$$c = (z_3 + z_7 - z_1 - z_9)/4g^2$$

$$d = (z_3 + z_6 + z_9 - z_1 - z_4 - z_7)/6g$$

$$e = (z_1 + z_2 + z_3 - z_7 - z_8 - z_9)/6g$$

$$f = (2(z_2 + z_4 + z_6 + z_8) - (z_1 + z_3 + z_7 + z_9) + 5z_5)/9$$

alt = f (metres, if z is in metres)

aspect = θ = arctan (e/d) degrees

gradient = arctan (d cos θ + e sin θ) = arctan (sqrt ($d^2 + e^2$)) degrees

$$\text{profc} = \frac{-200(ad^2 + be^2 + ced)}{(e^2+d^2)(1+d^2+e^2)^{3/2}} \quad \text{degrees /100m}$$

$$\text{planc} = \frac{200(bd^2 + ae^2 - cde)}{(e^2 + d^2)^{3/2}} \quad \text{degrees /100m}$$

OTHER FORMULAE FOR GRADIENT, translated into the same conventions :

Tobler (1969): arctan (sqrt $(z_2-z_8)^2 + (z_6-z_4)^2)/2g)$

Sharpnack and Akin (1969):

arctan (sqrt$((z_1+z_2+z_3-z_7-z_8-z_9)^2+(z_3+z_6+z_9-z_1-z_4-z_7)^2/6g)$

= arctan (sqrt ($d^2 + e^2$))

Struve (1977): as Tobler, except that (z_2-z_8) is replaced by $((z_1+z_3)-(z_7+z_9))$ if modulus of the latter is greater, and (z_6-z_4) is replaced by $((z_3+z_9)-(z_1+z_7))$ likewise and independently.

the parameter f defining altitude on the quadratic surface at the central point is used as 'calculated altitude' in place of the measured altitude, for comparison with the derivatives.

This technique produces a certain generalisation, which is desirable when altitude data are rounded and in error. Altitudes in the neighbourhood define trends in slope and the actual value is calculated for the point to which it is attributed. In the unlikely event of local surface shapes over the 3 x 3 neighbourhoods being exactly quadratic, we could claim to be calculating true point values. Since real land surfaces are more capricious, we will claim merely that the values calculated characterize the neighbourhood centred on the point to which they are attributed.

The surface model implied by this approach is that of Junkins, Miller and Jancaitis (1973). They showed how overlapping local polynomial surfaces could be combined with second order continuity by giving each an appropriate weighting field, declining smoothly from 1 at its centre to 0 at the edge of its square area of applicability. Since derivatives are calculated here for the centre of each such area, only one local quadratic is involved for each data point.

There is nothing pre-ordained about the use of a quadratic. It is the simplest full power series polynomial which provides the five characteristics of interest, while the full cubic would require more than nine data points and its extra characteristics (the possession of inflections) would not be used in the calculation of these derivatives. Likewise the 3 x 3 neighbourhood is the smallest permitting definition of all the relevant derivatives. If, however, the altitude data had a high error content, it might be advisable to smooth the data before processing: this amounts in effect to basing each quadratic on a broader neighbourhood, such as 5 x 5 or 7 x 7.

A limited check on the accuracy of these calculations was provided for the Thvera, Iceland, matrix by its originator, Jasbir Gill. For 75 randomly located points in the matrix, he manually determined gradient and aspect (as well as altitude). These values were compared with those calculated by fitting the local quadratic to a 3 x 3 neighbourhood of

a 100m-mesh grid. Comparison of calculated with measured altitude (Fig.3) shows that smoothing due to fitting of the quadratic produces very little error at the central point. Likewise the calculated gradients (Fig.4) and aspects (Fig.5) compare very closely with those estimated manually from contour spacing. Table 2 gives the corresponding least squares regressions (not constrained to pass through the origin) and confirms the small magnitude of the errors induced by transforming map to matrix and then fitting local quadratics. In each case, the regression coefficient is within 1.5 standard errors of its expected value of 1.0, and the standard error of the estimate is low in relation to the range of values encountered. In fact, on the first run of this comparison two outlying data points were found to reflect errors in the manual operation of measuring aspect, and not distortions due to matrix production and the fitting of local quadratics.

Table 2. Relationships between manually measured (M) and calculated (C) values of altitude, gradient and aspect at 75 points in the Thvera area, Iceland. r^2 = coefficient of determination, squared correlation coefficient: S.E.E. = standard error of the estimate of C : S.E.B. = standard error of the regression coefficient : S.D. = standard deviation of M.

	REGRESSION	r^2	S.E.E.	S.E.B.	S.D.
Altitude	C = 1.003M - 2.373	.99965	3.59m	.002	191m
Gradient	C = 0.984M + 1.416	.98404	1.27°	.015	10.1°
Aspect	C = 1.009M - 1.630	.99616	5.74°	.007	91°

The negligible loss of information suggests that gradient and aspect might as well be calculated from an altitude matrix (of this density), rather than measured laboriously from contour maps in a separate operation. Error appears to come mainly from the contour map itself, rather than from the further processing operations.

(2e) Zero-gradient points

In the tabulation of results for surface derivatives, another surface feature, which was not initially sought, forces itself on our attention. Sometimes gradient at the central point of a submatrix is exactly zero. In such cases the slope vector is vertically upward, so there is no aspect; neither is there any plan convexity. Since the results for such points are incomplete, the points are treated separately. Such special cases -singular points, surface-specific points - were analysed by Maxwell (1870), and interest in this topic has been rekindled by Warntz (1966,1967). Five such types of point are involved;

(i) Summits (peaks), which are higher than the immediate neighbourhood,

(ii) Pits, which are lower than the immediate neighbourhood,

(iii) Saddles (passes and pales), which lie at the self-crossing points of figure-of-eight shaped contours: they are minima on ridge lines and maxima on course lines,

(iv) Ridges, of interest here only if flat,

(v) Valleys, of interest here only if flat; and we must add a sixth case:

(vi) Plains, which are flat in any direction.

The zero gradient and hence non-existent aspect of such points is clear, but the convexity situations require further explanation. At summits and pits, contours are punctiform; lacking linear dimension, they have indeterminate curvature, so plan convexity is non-existent. It is nevertheless possible to measure surface curvature in two orientations, defining the maximum and minimum profile convexities (Report 5). For a saddle, one of these is positive and the other negative, but both are along slope lines and are therefore profile rather than plan convexities: the curvature of an X-shaped contour is indeterminate.

Valleys and ridges with zero gradient have profile convexity of zero in one orientation, and negative or positive respectively in the other. The zero convexity is regarded as a profile rather than a plan attribute since a small perturbation either way turns it into an indisputable profile convexity: if it takes a sign opposite to the other profile convexity, the point becomes a saddle, and if it takes the same sign, the valley becomes

a pit and the ridge becomes a summit; all these cases involve two profile convexities and indeterminate plan convexities. Likewise, if either of the zero convexities for a plain is perturbed, the point becomes a ridge or a valley.

Table 3. Discrimination of types of point where gradient is zero, from the maximum and minimum values of profile convexity, from Margaret Young (Report 5). These values are calculated from the quadratic coefficients a,b and c.

MAXIMUM $= -a-b+((a-b)^2 + c^2)^{1/2}$	MINIMUM $= -a-b-(a-b)^2+c^2)^{1/2}$	TYPE OF POINT
+	+	Summit
+	0	Ridge
+	–	Saddle
0	0	Plain
0	–	Valley
–	–	Pit

Table 3 shows how these different situations may be distinguished easily by calculating maximum and minimum values of profile convexity. This facility raises a question as to why other authors, such as Peucker and Douglas (1975), have made such heavy weather of automatically detecting such points from altitude matrices. The answer is in part, that they worked from the original data and not from a quadratic (which simplifies our consideration of curvature) and in part, that they asked different questions: they sought breaks of slope, and ridges and valleys with positive as well as zero gradient. The latter, however, does not explain the ridiculous results for pits and passes presented in their Figs.3(b) and 3(c), which suggest an error in programming the logical definitions they gave on p.377. Johnston and Rosenfeld (1975) and Grender (1976) produced more reasonable results, although their definitions of ridges and ravines (valleys) do seem rather crude.

Apart from the zero-gradient cases, it is difficult to distinguish ridges and valleys qualitatively from ordinary, curved slopes, Rather,

they grade into such slopes and can be distinguished quantitatively, by their stronger plan convexities (positive and negative, respectively) or by their opposed, lateral gradients considerably exceeding the gradient along their strike. In either case, an arbitrary threshold is required and horizontal continuity cannot be guaranteed if ridge or valley width varies: automated procedures currently 'look' at one scale at a time, while the human reader of a contour map contemplates a range of scales simultaneously and interpolates more continuous linear features.

It is now necessary to consider an initially paradoxical consideration. Summits, saddles and pits, at least, are singular points; any slight displacement from the point involved gives an ordinary slope. Why, then, do singular points coincide exactly on a number of occasions with the arbitrary points at which altitude is estimated and derivatives are calculated? Why should zero gradient occur exactly at the central point of the local quadratic?

This question is most easily answered by considering plains, which are by far the most common type of zero-gradient point. A plain, with no slope and zero convexity, occurs whenever all nine data points are identical. This situation would be highly improbably if all altitudes were independently measured to many significant digits, but we know this not to be the case. Altitudes are encoded with only a few digits each; the fewer the digits the less the number of distinct values possible, and the more frequent are 3 x 3 submatrices of identical values. Likewise a symmetrical distribution of altitudes around a central maximum gives a summit exactly at the central data point, whether we use a quadratic or any other means of interpolation within the 3 x 3 neighbourhood. A ridge (or valley) with zero gradient at the central point is produced if a row, column or diagonal of three identical values is symmetrically balanced. Hence the frequency of all types of zero-gradient point is a function not only of surface properties, but also of the resolution to which altitude is encoded. It would be interesting to test its senstiviity to further rounding of the input data.

For present purposes, this salutary result is of little concern since zero-gradient points are essentially nuisances, preventing complete tabulation of aspects and convexities. The conclusion is simply that excessive rounding of data values (e.g. to 10m) should be avoided. If, however, the detection and mapping of surface-specific points and lines was desired, it would be necessary to find all of them and not just those which happen to pass exactly through data points. This could be achieved by further analysis of the quadratic coefficients to locate, for example, any maximum, or any maximum closer to the central point than to any other data point. As noted above, identification of ridge and valley lines requires definition of an arbitrary threshold, probably of plan convexity: breaks of slope could be identified by a threshold of profile convexity, unaccompanied by beyond-threshold plan convexity.

For each zero-gradient point, the present program lists altitude, maximum and minimum values of profile convexity, and type of point, at the beginning of the output. Descriptive statistics are listed first with, then without such points. These points are excluded from correlations, scatter plots and histograms.

Chapter Three

CHARACTERIZATION OF AREAS

(3a) Introduction

The definitions of basic properties given in chapter two relate to points on the land surface. These points become small neighbourhoods by the exigencies of calculation (or of observation), but conceptually the derivatives are properties of any point on the land surface. This is considered the best starting point for general geomorphometry. The alternatives are to define some fundamental line (e.g. slope profile) or area (e.g. facet) on the surface; we have seen in section (2c) that there are problems in achieving agreement on the definition of either, and that neither approach gives a balanced, unbiased sampling of the land surface.

When starting from point values, an area is characterized by statistical and spatial distributions, and by simple and multiple relationships between the values. Hence frequency distributions, maps and within-area relationships will be discussed in turn, and decisions taken in specifying the computer operations outlined in Report 5 will be justified. This will be followed by a brief consideration of between-area relationships, and finally the important effect of grid mesh on characterizations of areas will be discussed. Each section will contain some discussion of results, to illustrate more fully their meaning and interpretation.

(3b) Frequency distributions and possible transformations

The elementary statistical approach to such data sets is to abstract values from their spatial context and group them on their scale of magnitude, giving a frequency distribution. The principal form of presentation of such a distribution should be a histogram, a plot of frequency for classes of (preferably) equal width on the magnitude scale. A well-designed histogram gives a rapid visual impression of distribution shape, as well as location and spread on the magnitude scale. It should be supplemented by numerical summaries, but the possibilities of outliers and of multimodality make it unsafe to rely entirely on such summaries : the histogram should always be consulted.

Sturges (1926; see also Huntsberger and Billingsley, 1977) has proposed a rule of thumb that the number of classes in a histogram should ideally be: $1 + \log_2 N$, where N is the number of observations: if the frequency distribution is Gaussian, the class frequencies will be a binomial series. A second suggestion is $5 \log_{10} N$ classes (Brooks and Carruthers, 1953 p.13). Neither of these seems to be of any value; at least, they do not extrapolate to samples of the size considered here. Indeed, their authors were concerned with much smaller samples. I have found, in connection with work both on this project and on the G.B. census, that division of 10,000 (or even of 2,000) values into 100 classes produces a clearer and more informative view of the distribution than division into fewer, coarser classes. If this information can be portrayed clearly, which is possible even with a line printer, there is no reason to submerge it in more generalised classes. (The above rules suggest only 14 or 20 classes for 10,000 values; 12 or 16 for 2,000). Doane's (1976) proposed modification of Sturges' Rule (by adding extra classes to allow for skewness) does nothing to reduce my objection to its whole basis; this rule ignores the realities of applied statistics.

Moreover, it is demonstrated in the next two chapters that fine subdivision of a frequency distribution may reveal interesting defects in data quality, which become apparent from multiple modes, or from gaps because certain values are impossible. Indeed, for this purpose it might be useful to have 'no-class histograms' or dispersion plots, where spacings on the magnitude scale are shown in exact detail, regardless of the number of values. (See also the 'no-class' statistical map debate : Tobler, 1973; Dobson, 1973; Evans, 1977). The presence of real polymodality in slope gradients continues to excite discussion among geomorphologists (Statham, 1975; Carson, 1977) but the question of significance of subsidiary modes remains elusive, especially with the sampling problems

noted above. It was decided to use up to 110 classes, this being ten fewer than the number that will fit onto two line printer pages at 60 lines/page, 1 line/class. (Frequency is shown horizontally by the number of symbols per line; on a computer line printer it is much more convenient to produce variable-length lines horizontally rather than vertically). Where the range of variation is unpredictable, as for altitude, it is necessary to program the calculation of class size. Some programs simply divide the range by the desired number of classes, but this gives awkward numbers for class limits and midpoints. Instead, for altitude, a class width of 1m is used for ranges in altitude of up to 110m, 2m is used up to 220m, 5m up to 550m, then 10m class widths and so on with the decimal place shifted. This gives at least 44 classes (unless range in altitude is less than 44m) and never more than 110 classes. It provides round-numbered class limits (See Doane, 1976, for a definition of 'roundness'), and it works very well for the data processed to date (Figs. 18,31).

Other variables are more predictable in value. Aspect usually ranges over the full scale from 0 to 360 degrees : for almost any area, a class width of 5 degrees gives 72 classes. Gradient varies from 0 to 90 degrees, so a class width of one degree was chosen. In practice, the upper half of the range is rarely encountered, so a width of half a degree might be even better. However, because of the importance of small differences in the low-gradient range, a second, 100-class histogram is produced with classes 0.1 degrees wide for the range 0 to 10 degrees. This enlargement is very valuable in judging data quality : if altitudes are excessively rounded, only a discrete number of low gradients will be possible, and these show up as spikes separated by vacant classes.

Both profile and plan convexity have long-tailed distributions, more or less centred on zero, but initially their ranges were unpredictable. By trial and error, it was found that almost all observations fell between -50 and +50 degrees per 100m for profile convexity, and -500 and +500 for plan convexity. It was therefore convenient to use class widths of 1 and 10 respectively, giving 100 classes in each case. Fewer classes would obscure the shape of the relatively narrow frequency peak. The number of values lying beyond each extreme is printed beneath the histogram.

Of the widely available programs, S.P.S.S. produces terrible histograms; for some unknown reason, it separates each horizontal bar of a histogram by four blank lines. A locally written program was available, but it was found more convenient to call the MIDAS package program. To the left of each frequency bar this gives first the count (frequency) for that class, then the percentage of values in the histogram falling in that class, and finally the class midpoint (with an excessive number of zeroes, to allow for class widths less rounded than ours). Class limits are included in the class above. The first and last classes are half the width of the others, and the last (highest) class includes its upper limit. The major defect of this program is the absence of cumulated % frequencies, which are needed for percentile-based statistics. The number of cases per cross in the symbolised bar is printed at the top of the histogram, having been selected automatically so that the line printer paper width is not exceeded. The last, however, represents the remainder : if the scale claims 'each X = 3', one X represents either 1, 2 or 3 cases, and 10 is represented by four Xs.

The (visual) symmetry of histograms for both types of convexity justifies the definitions used. If signs were ignored so that one limb was 'folded over' onto the other, or if the positive and negative halves were considered separately they would be J-shaped, and moment statistics would be difficult to interpret.

Numerical summary of the results, following Evans (1972), is in terms of the moment measures mean, standard deviation (RMS), skewness and kurtosis. These are based respectively on the first, second, third and fourth powers of deviations about the mean (or about zero, for the mean itself). The first two are in the same units as the original values, but skewness and kurtosis are standardised to be dimensionless measures of distribution shape. Because only even (second and fourth) powers enter into the calculation of crude kurtosis, only positive values are possible; a Gaussian distribution has a crude kurtosis of 3.0. We have followed the convention of subtracting 3 from the initial value for kurtosis, so that the Gaussian expectation is zero, as for skewness. However, skewness ranges from minus to plus infinity, while kurtosis thus modified ranges from -3.0 to plus infinity. Positive kurtosis indicates that the peak and/or tails of the distribution are more prominent than in a normal distribution; negative, that the peak is broad and/or the tails underdeveloped (Finucan, 1964). Skewness indicates which tail is the more prominent, the positive or the negative.

The moments are calculated by the terrain analysis program OMY8 but the formulae used are the same as in S.P.S.S. (Nie et al., 1975) and it has been checked that the results are identical. Maximum and minimum values are printed in the same table, since they are often useful in checking data and in the interpretation of higher moments.

Zero gradient points, as discussed in section (2e), have to be treated specially because they lack a full set of descriptors. Hence they are omitted completely from the first summary table (corresponding to deletion of incomplete cases). They are included, wherever possible, in a second summary table on the same page, below the correlation matrix. Table 4 exemplifies this page of summary statistics.

The adoption of skewness and kurtosis as routine descriptors implies that there is no easy and acceptable way of transforming these frequency distributions into approximately Gaussian (normal) distributions (Mosteller and Tukey, 1977, ch.5; Kruskal, 1968). This was investigated in Report 4 for gradient frequency distributions generated in four quite different ways. It was concluded that, even within one type of data, different transformations were required for gradients in different areas. This is even more likely to hold for altitude. The most useful transformations for gradient are the logarithm of tangent (Speight, 1971) and the square root of sine, but some homogeneous data sets such as that for the Ferro basin are best left untransformed. It will be shown that geomorphic interpretations are available for skewness and kurtosis; at present, it seems best to use these rather than perform transformations which do not normalise the data.

Moment descriptors are not appropriate for aspect, which has a circular scale; its conventional expression in degrees east (clockwise) from north masks the fact that its extremes are coincident ($0 \equiv 360^{\circ}$). Central tendency in aspect is measured by the direction of the resultant vector, and the strength of this tendency (an inverse measure of dispersion) is given by the vector strength, i.e. the length of the resultant expressed as a proportion of the total length of the vectors summed (Curray, 1956: see also Mardia, 1972). Both measures are printed below the summary table, giving equal weight to each observation of aspect (unit vector). But it was noted above that the

Table 4 . Summary statistics page of computer output for the Thvera matrix

```
THVERA, N-CENTRAL ICELAND
 NO. OF ROWS=  102
STATISTICS FOR         9993POINTS WITH NON ZERO GRADIENT
          EST.ALT.     GRADIENT  PROFC      PLANC

MEAN       881.390     21.366    -0.094     0.427

SDEV       229.572     10.506    10.130    59.887

SKEW        -0.540      0.122     2.252     6.338

KURT        -0.269     -1.071    10.916   201.499

 MAX      1368.333     51.269    99.829  1789.373

 MIN       262.778      0.477   -79.383 -1145.916

 VECTOR MEAN ASPECT ANGLE               49.215
 VECTOR STRENGTH(PROPORTION)             0.085
 GRADIENT WEIGHTED VECTOR MEAN ASPECT ANGLE        55.770
 GRADIENT WEIGHTED VECTOR STRENGTH(PROPORTION)      0.021

CORRELATION COEFFS

             EST.ALT.     GRADIENT        PROFC        PLANC

EST.ALT.        1.000        0.119        0.346        0.206

GRADIENT        0.119        1.000       -0.063        0.016

PROFC           0.346       -0.063        1.000        0.174

PLANC           0.206        0.016        0.174        1.000


STATISTICS INCLUDING ZERO GRADIENT POINTS
EST ALT AND GRADIENT FOR ALL      10000   POINTS
PROFC AND PLANC FOR         9999   NON ZERO AND PLAIN POINTS
WHERE PLANC IS TAKEN AS 0.0 FOR PLAIN POINTS
         EST.ALT.     GRADIENT  PROFC      PLANC

MEAN      881.453     21.352    -0.094     0.426

SDEV      229.740     10.517    10.127    59.869

SKEW       -0.540      0.120     2.253     6.340

KURT       -0.270     -1.068    10.925   201.622
EXECUTION TERMINATED    18:50:02  T=39.267  RC=0        $4.45
T=39.479 CR=0


$R CER2:GMYL 7=GGM1:GECD
EXECUTION BEGINS    18:50:03
EXECUTION TERMINATED    18:50:04  T=.049  RC=0          $.01
```

importance of aspect increases with gradient. Hence a second, weighted calculation is made in which the length of each input vector is proportional to gradient in degrees, and this is printed next. The difference between the two pairs of results is often considerable; weighting produces a great reduction in strength. Asymmetry due essentially to gentle slopes is of suspect importance, and often reflects only the delimitation of the study area.

Table 5 gives, in readily compared form, the summary statistics for eight matrices; three in the Cache area, three for square areas in glaciated mountains, and two (Ferro and Gold Creek) for fluvial drainage basins. These statistics will be reviewed briefly, in order to elucidate their geomorphic meaning.

Table 5. Summary statistics for the eight matrices analysed to date. Mean, standard deviation (SD), skewness, kurtosis, maximum and minimum are given for each property except aspect. For aspect, the direction ('mean') and strength of the resultant vector are given with and without weighting by gradient. Mesh, vertical resolution and size of data set are also given.

AREA	MEAN	SD	SKEW	KURT	MAX	MIN
			ALTITUDE			
Cache 1	379	5.5	+.426	-.44	398	367
Cache 2	404	22.9	+1.779	+2.31	483	379
Cache 3	439	24.7	+ .580	-.49	526	403
Torridon	445	152.6	+ .728	+.21	980	33
Thvera	881	229.6	- .540	-.27	1368	263
Nupur	449	191.9	- .246	-.96	776	10
Ferro	445	211.4	+ .392	-.12	1148	8
Gold Cr.	666	15.8	- .191	-1.50	692	639
			GRADIENT			
Cache 1	1.56	1.01	+2.92	+19.44	15.29	.38
Cache 2	3.45	3.88	+1.83	+2.58	23.12	.38
Cache 3	5.84	3.70	+ .46	- .53	21.67	.38
Torridon	14.76	11.09	+1.01	+ .15	54.85	.58
Thvera	21.37	10.51	+ .12	-1.07	51.27	.48
Nupur	21.87	12.43	+ .19	- .97	59.14	.48
Ferro	13.07	5.04	+ .04	+ .21	35.00	.95
Gold Cr.	4.95	2.65	+1.45	+3.35	20.32	.14
			PROFILE CONVEXITY			
Cache 1	-.15	6.99	+ .74	+38.25	112.5	-116.3
Cache 2	-.36	6.79	+ .08	+ 2.26	38.8	- 39.3
Cache 3	-.60	8.55	+ .17	+ 1.93	44.8	- 41.8
Torridon	-.88	8.45	+1.14	+12.48	73.2	- 76.8
Thvera	-.09	10.13	+2.25	+10.92	99.8	- 79.4
Nupur	+.39	11.89	+1.31	+ 9.03	112.2	-116.6
Ferro	-.61	6.54	+ .06	+11.69	77.7	- 88.6
Gold Cr.	+1.42	23.45	-1.08	+14.88	83.1	- 43.3

Table 5 (continued)

PLAN CONVEXITY

AREA	MEAN	SD	SKEW	KURT	MAX	MIN
Cache 1	+7.18	281.6	+ .48	13.4	1833	-1833
Cache 2	+4.73	218.1	+ .32	21.7	1833	-9167
Cache 3	+8.64	201.5	+6.80	173.3	6417	-1833
Torridon	+1.92	60.1	-12.97	818.3	1031	-3123
Thvera	+ .43	59.9	+6.34	201.5	1789	-1146
Nupur	- .67	52.7	+2.27	60.7	999	- 688
Ferro	- .81	70.8	-1.52	207.9	1833	-2063
Gold Cr.	+40.52	519.4	+26.58	1177.9	23354	-2635

	ASPECT				NUMBERS OF:				
	Unit vectors		Gradient-weighted						
	MEAN	STRENGTH	MEAN	STRENGTH	INCLUDED POINTS	PLAINS	OTHER ZERO GRADIENTS	GRID MESH	VERTICAL RESOLUTION
Cache 1	242	.242	239	.007	8540	1011	53	25m	1m
Cache 2	243	.412	242	.021	8983	595	26	25m	1m
Cache 3	329	.200	343	.021	9481	110	13	25m	1m
Torridon	015	.180	004	.017	9372	227	5	100m	1m
Thvera	049	.085	096	.021	9993	6	1	100m	5m
Nupur	095	.113	079	.044	6013	70	1	100m	5m
Ferro	081	.126	076	.029	11582	211	3	100m	10m
Gold Cr.	076	.530	074	.049	3447	0	0	7.62m	.03m

Mean altitude is important mainly as a descriptor of location, and has no direct implication as to land surface form except that (on land) a low mean is unlikely to be accompanied by high variability. The latter is measured by standard deviation, which Evans (1972) recommended as a more stable alternative to the commonly used range of altitude (relief); Mark (1975a) still preferred the range. Standard deviations are high not only for the three glaciated mountain areas, but also for the Ferro basin (see chapter 4, below). The small Gold Creek basin (near Canberra, Australia) and the Cache areas (see Chapter 5, below) have much less variability in altitude, though even Cache 1 is not a plain.

Skewness of altitude reflects the relative lengths of the upper and lower tails of the frequency distribution. If the mode is high relative to the range of values (cf. a plateau area), skewness is usually negative; if it is low (cf. an inselberg area),positive skew is expected. The positive skew of Cache represents the extent of lowland among the mountains, while that of Torridon is due to the breadth of glacial trough floors, exceeding that of mountain tops. For different reasons, Ferro has a broad alluvial valley floor while its divides are sharp, and this gives mild positive skew. Nupur and Thvera have areas of flat (basaltic lava) summit plateaus; the cirque floors are relatively high, and few valley floors are incorporated, so altitudes have small negative skews. None of the altitude statistics distinguishes glaciated from other mountains.

Kurtosis of altitude is weakly negative in almost all cases, reflecting the absence of long tails in the altitude frequency distributions. This in turn reflects the strong positive spatial autocorrelation of altitude. The one exception is Cache 2, where the (positive) tail comes from mixture of a small upland with an area which is dominantly lowland.

Mean gradients increase in line with variability in altitude, except that Ferro is not so steep as the glaciated mountains, and its gradients are much less variable. Standard deviation of gradient relates mainly to mean gradient, except that again Cache 2 is more variable than expected. Skewness of gradient is positive in all cases, especially those with low mean gradients where the lower limit of zero is much closer than the upper limit of 90 degrees. Ferro, Thvera and Nupur are almost unskewed, but only Ferro approximates a normal distribution; the others have very broad modes

and short tails, hence negative kurtosis. Torridon differs from the other two glaciated mountain areas in having normal kurtosis and positive skew; like its positively skewed altitude, this may reflect the extent of valley floors. Otherwise (e.g. for Cache) skewness in excess of 1.0 produces positive kurtosis.

Convexity statistics are more difficult to interpret, and the reader is referred to later chapters for discussion of Cache and Ferro. Both for profile and for plan, mean values should approximate to zero, since convexities will be balanced by concavities except in limited areas or in exceptional topographies. This is indeed the case (the Gold Creek basin covers a very small area) and no useful generalisations can be made for example about mean values for the three glaciated mountain areas.

Standard deviations of convexity express the general magnitude of curvature, with convexity and concavity undistinguished. The results are consistent for the eight areas studied, except that (i) values for Gold Creek are remarkably high, both in profile and in plan; and (ii) plan convexities are much higher for Cache than for Ferro and the glaciated mountain areas - this is probably due to data problems (Chapter 5). Otherwise, standard deviations are between 6.5 and 12.0 degrees per 100m for profile convexity, and between 50 and 71 for plan convexity. The exceptionally high values for Gold Creek may be a true reflection of the much finer grid mesh (7.62m) and vertical resolution (.03m).

A remarkable consistency is that skewness of profile convexity is positive for all areas except Gold Creek. For the glaciated mountains it exceeds +1.0. This suggests that frequent gentle concavities are balanced by fewer but more extreme convexities, a 'polyconcave' surface form which I have predicted should be found where cirques and arêtes are common.

This comparison between convexities and concavities for glaciated mountains is presented in Table 6, which affirms the impression given by skewness. Taking profile curvatures in excess of $40.5^{\circ}/100m$, convexities outnumber concavities by 34 to 11 in Torridon, by 53 to 4 in Nupur, and by a remarkable 88 to 1 in Thvera. Obviously ridges are much sharper than valleys in all three areas. The sequence Torridon : Nupur : Thvera is also a sequence of increasing positive skew, and relates to a map interpretation of increasing degree of glacial dissection. The marked imbalance continues through the next 10° class of curvature (especially for Thvera), until at around $19^{\circ}/100m$ there is a

Table 6. Summarised frequency distributions of convexity for the three glaciated mountain areas. Numbers of convexities (+) and concavities (-) are juxtaposed, for each magnitude range (degrees per 100m in each case).

PROFILE CONVEXITY

	Torridon		Thvera		Nupur	
	-	+	-	+	-	+
above 50.0	9	18	1	41	2	26
40.5 to 50.0	2	16	0	47	2	27
30.5 to 40.5	13	44	0	107	11	117
20.5 to 30.5	58	105	30	219	59	201
10.5 to 20.5	580	441	784	507	602	417
1.5 to 10.5	3964	1994	4109	2209	2229	1385
0.5 to 1.5	776	620	692	543	330	281
totals	5402	3238	5616	3673	3235	2454
+ 0.5 to -0.5	732		704		324	
overall totals	9372		9993		6013	
median	-1.423		-1.395		-1.192	
mean	-.877		-.094		+.391	
skew	+1.145		+2.252		+1.313	

PLAN CONVEXITY

	Torridon		Thvera		Nupur	
	-	+	-	+	-	+
above 500	2	9	5	19	2	6
405 to 500	5	4	2	9	3	3
305 to 405	6	7	2	11	6	5
205 to 305	24	31	18	35	16	24
105 to 205	129	107	75	137	78	77
15 to 105	2010	2344	2138	1360	1377	1215
5 to 15	1222	1534	1972	1485	915	982
totals	3398	4036	4212	3056	2397	2312
+5 to -5	1938		2725		1304	
overall totals	9372		9993		6013	
median	+.165		-.212		-.033	
mean	+1.924		+.427		-.673	
skew	-12.973		+6.338		+2.271	

turning point, below which concavities outnumber convexities. All medians are negative, but their order does not agree with that based on skewness, on the extremes and on the map interpretations.

The corresponding plan convexities do not present such a clear picture; indeed, those for Torridon have a strong negative skew despite convexities outnumbering concavities (in excess of $500^{\circ}/100m$) by 9 to 2. The other two, however, show positive skew, negative medians, and a preponderance of convexities which for Thvera extends down to $105^{\circ}/100m$ but for Nupur (as for Torridon) only applies in excess of $500^{\circ}/100m$. Thvera is again the most extreme. The 'polyconcavity' of a cirque-and-trough landscape implies sharper convexity in plan as well as profile, but it appears that this is apparent first in profile convexity, and only shows in plan convexity for areas very well dissected by local glaciers.

Kurtosis of convexity is strongly positive in all cases because of the long tails. No particular pattern can be discerned in its relative magnitude, which is very strongly influenced by extreme points.

The use of quantile-based statistics instead of moments might be worth investigation, but it might hide interesting differences in the extreme tails such as those discussed for the convexity of glaciated mountains. The moment-based statistics are generally satisfactory except that non-zero skewness produces a bias to positive kurtosis. Indeed, (positive) skewness and kurtosis are positively correlated in many families of theoretical frequency distributions. Kurtosis might be a more useful, independent statistic if it were measured after a transformation to zero skewness (Box and Cox, 1964). In this way, the relationship between skewness and kurtosis could be removed and kurtosis interpreted as the relative importance of the tails at zero skew.

The value of summary statistics could be summarised by describing the glaciated mountain areas as having high mean and variability of gradient, high variability of altitude, positive skew of profile convexity and an excess of strong positive plan convexities.

Vector mean aspects are around 230° or 330° for Cache (but see chapter 5c), and northward or eastward for the other matrices. Gold Creek and Cache 2 have quite strong directional tendencies when each observation is given unit weight. Like all the others, however, this tendency is very weak - only a few percent - after weighting by gradient. The remarkable thing is that this pronounced weakening is not accompanied by much of a change in the vector mean. This changes most (by 47°) for Thvera, otherwise by only 1° to 16°.

(3c) Replicability

If these statistics are to be interpreted as characteristic of topography within a broader region, it is interesting to know how they vary when the square 'sample area' is displaced. Given the cost of replicating sample areas, the easiest way to obtain some idea of this variation is to subdivide a sample area and repeat all calculations for each sub-area. This, indeed, was done for the Cache area, but with the deliberate intention of studying variation between an upland and a lowland area (Chapter 5).

The Thvera altitude matrix is large enough (102 x 102) to be subdivided into four quadrants; northwest, northeast, southwest and southeast. Although these quadrants are not too dissimilar in appearance on the topographic map, it must be remembered that their smaller dimensions mean that they may not be large enough to provide replicable statistics. With this major qualification in mind, we may ask the question: 'which statistics are stable, and which vary excessively from one quadrant to another?'

Table 6a gives the statistics to be compared. Altitude has the most stable frequency distribution. In this area, the maximum is more stable than the mean; skewness and kurtosis are consistently negative, with kurtosis the more variable. For gradient, the standard deviation and (near-zero) minimum are most consistent : kurtosis is negative throughout, but skewness is unstable. For the convexities, standard deviations are fairly stable and kurtosis is strongly positive but variable; skewness is consistently positive only for profile convexity. Maxima are variable and minima are quite unstable. Means oscillate around zero. None of the aspect statistics is stable, but weighting by gradient reduces vector strength to less than 40% of the unweighted value.

The most stable correlations are those between altitude and the convexities, especially profile. Correlations of profile convexity with plan convexity and (negatively) with gradient are moderately consistent, but other correlations are negligible or unstable. The positive correlation between altitude and gradient varies from .06 to .30.

Table 6a Summary statistics and linear correlations for the four quadrants of the Thvera matrix, N.C. Iceland. Mean, standard deviation (SD), skewness, kurtosis, maximum and minimum are given for each property except aspect. For aspect, the direction ('mean') and strength of the resultant vector are given with and without weighting by gradient. Supplied by J.S. Gill.

ALTITUDE

QUADRANT	MEAN	SD	SKEW	KURT	MAX	MIN
NW	863	213	-.236	-.421	1318	359
NE	819	239	-.502	-.600	1306	263
SW	992	150	-.362	-.204	1356	541
SE	840	262	-.238	-.822	1368	310
RANGE	173	112	.266	.618	62	278

GRADIENT

QUADRANT	MEAN	SD	SKEW	KURT	MAX	MIN
NW	21.4	10.5	.203	-1.037	51.3	0.7
NE	24.3	9.9	-.239	-1.003	49.6	0.7
SW	17.1	9.6	.641	-.387	48.2	0.5
SE	22.2	10.6	-.019	-1.076	46.5	0.7
RANGE	7.2	1.0	.880	.689	4.8	0.2

PROFC

QUADRANT	MEAN	SD	SKEW	KURT	MAX	MIN
NW	.08	10.9	2.137	8.053	73.7	-26.2
NE	-.04	11.0	2.613	14.304	95.6	-79.4
SW	.15	9.0	1.599	5.833	64.7	-27.4
SE	-.45	9.2	2.341	13.098	99.8	-24.7
RANGE	.60	2.0	1.014	8.471	35.1	54.7

PLANC

QUADRANT	MEAN	SD	SKEW	KURT	MAX	MIN
NW	2.15	70.1	13.323	272.095	1789	- 286
NE	- .44	65.3	.754	128.860	1077	-1146
SW	-1.11	50.0	-1.450	153.768	688	-1146
SE	.09	50.8	5.525	91.409	993	- 425
RANGE	3.26	20.1	14.773	180.686	1101	860

ASPECT

	Unit vectors		Gradient-weighted	
	MEAN	STRENGTH	MEAN	STRENGTH
NW	002.8	.264	342.1	.070
NE	072.7	.128	105.0	.048
SW	262.1	.108	211.4	.024
SE	104.2	.208	123.1	.070
RANGE	202.1	.156	(229.3)	.066

CORRELATIONS

	ALT:GRAD	ALT:PROFC	ALT:PLANC	GRAD:PROFC	GRAD:PLANC	PROFC:PLANC
NW	.195	.369	.191	-.070	-.028	.094
NE	.190	.338	.228	-.059	.062	.214
SW	.061	.363	.251	-.081	.022	.154
SE	.302	.366	.226	-.025	.029	.226
RANGE	.241	.031	.060	.056	.040	.132

Most of the tendencies considered noteworthy above, for the whole matrix, apply to each of the quadrants. Gradient has various skews for the quadrants, but little skew overall, and the positive skew of plan convexity is based essentially on two quadrants. Mean gradient and standard deviation of altitude are perhaps rather more variable than expected. On the whole, however, the similarity of land surface form in four quadrants of Thvera is supported by these statistics.

A different type of replication is possible when 'thinned-out'grids of 200m or 300m mesh are analysed. Starting from 100m grid, there are four possible 200m grids and nine possible 300m grids. The ranges of statistics produced for the central half (40 x 80) of the Nupur matrix are given in Table 6b, together with the four sets of statistics for 200m (samples A...D); the nine sets of 300m results would take too much space. Whereas the previous table considered spatial replication between different but adjacent sample areas, this table presents the results of repeated sampling within the same area. It thus provides some idea of the inherent variability in the method for this (coarse) grid mesh and this (smaller than usual) sample size, whereas Table 6a also incorporates spatial variation.

As expected, Table 6b shows much more stability than Table 6a. Altitude and gradient statistics are extremely stable; the four 200m values for mean gradient range through only 0.4% of their mean value. Even the weak positive skew and negative kurtosis of gradient are stable. 300m statistics for altitude and gradient have ranges about twice as great as 200m values. This is not the case for profile convexity, and plan convexity is most unstable for 200m, even though extreme values are equally frequent at 300m. But the plan convexity statistics are extremely unstable throughout: apparently a sample of 740 is inadequate for this long-tailed frequency distribution.

Profile convexity statistics are reasonably stable except for kurtosis, and even that is consistently positive both at 200m and at 300m. Also at both scales, maximum profile convexity is more extreme than minimum, i.e. convexities are sharper than concavities. Aspect statistics are now consistent throughout, quite unlike Table 6a; despite the weak vector strengths, the mean is eastward with or without gradient-weighting.

Table 6b Summary statistics and linear correlations for four 200m-mesh samples of the central half of the Nupur matrix, N.W. Iceland, each displaced 100m from the others in the original 100m-mesh data. Mean, standard deviation (SD), skewness, kurtosis, maximum and minimum are given for each property except aspect. For aspect, the direction ('mean') and strength of the resultant vector are given with and without weighting for gradient. Each 200m sample provides 740 values of the derivatives. The nine 300m mesh samples of the same area each provide 300 values. Ranges of the resulting statistics are given for comparison with the 200m ranges. Supplied by J.S. Gill.

ALTITUDE

SAMPLE	MEAN	SD	SKEW	KURT	MAX	MIN
A	478	189.8	-.381	-.87	...	48
B	472	194.1	-.367	-.88	786	35
C	477	190.9	-.365	-.89	772	48
D	472	195.1	-.357	-.91	778	34
200m RANGE	6	5.3	.024	.04	14?	14
300m RANGE	13	9.2	.047	.05	39	28

GRADIENT

SAMPLE	MEAN	SD	SKEW	KURT	MAX	MIN
A	19.43	9.55	+.118	-.71	...	.34
B	19.40	9.63	+.118	-.83	42.6	.34
C	19.35	9.47	+.095	-.83	43.7	.34
D	19.36	9.54	+.092	-.81	43.0	.34
200m RANGE	.08	.16	.026	.04	1.1?	.00
300m RANGE	.29	.31	.127	.21	3.2	1.19

PROFC

SAMPLE	MEAN	SD	SKEW	KURT	MAX	MIN
A	+.671	6.81	+.85	+.77	...	-14.0
B	+.557	6.62	+.72	+.26	24.9	-13.4
C	+.630	7.05	+.96	+1.76	38.8	-16.8
D	...	6.91	+.78	+.86	25.6	-21.2
200m RANGE	.114?	.43	.24	1.50	13.9?	7.8
300m RANGE	.221	.23	.41	.89	6.6	.9

PLANC

SAMPLE	MEAN	SD	SKEW	KURT	MAX	MIN
A	+6.06	187.7	+26.14	+698.43	...	-222
B	+ .68	37.5	+ 1.56	+ 30.85	317	-359
C	+ .19	25.9	+ .63	+ 6.19	166	-131
D	...	39.2	+ 7.46	+116.36	662	-147
200m RANGE	5.87?	161.8	25.51	692.24	496?	228
300m RANGE	4.62	28.3	17.2	156.01	300	712

ASPECT

	Unit Vectors		Gradient-weighted	
	MEAN	STRENGTH	MEAN	STRENGTH
A	101	.135	087	.048
B	100	.149	088	.055
C	109	.128	093	.048
D	106	.152	093	.054
200m RANGE	009	.024	006	.007
300m RANGE	010	.042	017	.011

CORRELATIONS

	ALT:GRAD	ALT:PROFC	ALT:PLANC	GRAD:PROFC	GRAD:PLANC	PROFC:PLANC
A	-.052	.605	.078	-.120	-.062	.014
B	-.038	.610	.246	-.120	-.004	.119
C	-.062	.604	.290	-.119	.015	.219
D	-.051	.605	.233	-.112	-.023	.129
200m RANGE	.024	.006	.212	.008	.058	.205
300m RANGE	.070	.041	.153	.069	.205	.209

The most remarkably consistent correlation is the strongest, that between altitude and profile convexity. Correlations involving plan convexity are unstable, probably because of the influence of extreme values. Others, however, are consistent in sign even when very weak. In general these results support use of the statistics chosen, except for plan convexity where further studies of replicability, extreme values, and sample size are required.

(3d) Maps

Although it is very necessary to use frequency distributions and related numerical summaries when describing an area, their utility is limited in that values are abstracted from their geographic positions. Since relative positions are highly important in understanding patterns in the data, frequency distributions must be complemented by maps. Indeed, many of the characteristics of frequency distributions are easily interpreted only by reference to the corresponding maps. Maps are also desired as an end-product in many applications.

It is now feasible to produce high-quality maps from specialised computer output devices, but this involves joining a different job queue, one which usually moves more slowly than that for the line printer. The first priority, then, is to produce maps on the line printer to accompany the histograms and statistics. Further maps can be produced later if the files of derivatives are stored, and they can be designed to take account of characteristics portrayed by the line printer maps. For example, maps of gradient and of profile convexity have been produced on a larger drum pen plotter, with contouring by the GPCP package. Convexity surfaces, however, are extremely rugged, with contour lines which are closely spaced and too complex for easy interpretation.

For the density of information to be portrayed here, it is best to produce a shaded symbol for each value; with such data, an attempt to produce contour lines on the line printer would result in chaos. The production of acceptable shading is easy given a line printer chain with special characters (Coppock, 197[illegible]) or modern equipment such as the electrostatic printer/plotters produced by Versatec. Unfortunately, neither is yet available in Durham. Difficulties encountered in producing a scale of intensities of shading using ordinary line printer characters were discussed, for example, by the Experimental Cartography Unit (1971). The main problem is to avoid an 'inversion' in the scale, whereby a lower class appears darker than a higher one, as when all ten classes of the widely used SYMAP program default are used: the resulting synoptic view is misleading and the symbols must be interpreted qualitatively, class by class. The surest way of avoiding an inversion is by overprinting an extra character

for every higher class (Siccama, 1972).

With ordinary line-printer symbols, it is desirable to use a small number of classes. This number should be even if the mean value is to be used as a class limit, a central reference point. Hence six classes were chosen, with a single overprint for the fourth and fifth, and a triple overprint for the sixth (darkest) class, using the following symbols:

. + O O- OX OXAV

The symbols are chosen to be as symmetrical as possible, avoiding perceptually distressing linearity. The one problem is that although the sequence of increasing darkness seems clear-cut on a typewriter, in practice the frequent use of full stops on line printers causes the symbol to spread, becoming as dark as the + or even the O. For similar reasons, darkness may vary across the page. The resulting maps (Figs. 23, 25-27, 36, 38-40) show good distinction in darkness between the top four classes: the darkness of the three lowest classes are too similar for rapid discrimination, but these classes can readily be distinguished qualitatively. Blank cannot be used for a class since excluded and zero-gradient points are left blank.

Using a line printer, there is a maximum line width which varies between installations: lines up to 125 characters long are generated by the present program. For data sets with more than 125 columns, it is necessary either to compress the results so that one character represents more than one value, or to split maps into several sheets. Neither procedure is agreeable; the second is adopted here. A new map sheet is started with the 126th column, a third with the 251st column, and so on. Up to 10 sheets are possible and they follow each other in sequence, each starting on a new page of the line printer output. The length of each sheet is limited only by the number of pages of output requested. Using one symbol per data element, the only data set among the eight considered here to require a second map sheet was Ferro, which is an elongate drainage basin.

On line printers, the spacing of characters along a line (.1 inch; 2.54 mm) does not equal the spacing of lines (usually .167 or .125 inch; 4.24 or 3.17mm).

Hence each value in the square grid is allotted a rectangular printing position instead of a square one. The resulting elongation of the map can be avoided either by printing 5 x 3 or 5 x 4 blocks to symbolise each value, which makes the maps five times as wide as they need be; or by interpolating extra columns (MacDougall, 1976), which ends the highly desirable one-to-one symbolisation of data elements in non-interpolated maps. The former solution is very clumsy for data sets of the present size, and exaggerates the problem that a limited number of characters are available per row.

Clearly the number of separate sheets should be minimised, and this is taken as a good reason not to attempt squaring of the rectangular line printer output. Line printer maps are accepted as rough working tools and, given their general coarseness, the further degradation caused by elongation is not excessive. Fortunately, economy measures at our NUMAC computer centre led to adoption of the closer line spacing; hence the symbol spaces of the line printer maps in this Report are only modestly elongated at 2.54 x 3.17mm (before photo-reduction). These line printer problems could be solved by a modest investment in special-purpose equipment, but they typify the present computing environment in Durham and in many other centres.

The same six-class grey scale is applied to maps of altitude, gradient, profile convexity and plan convexity. The next step is to choose a set of class intervals so that internal variability is clearly portrayed. The principles involved in selection of class limits have been thoroughly reviewed elsewhere (Evans,1977a) : here we select a central class limit (zero or the mean), then assign the other limits so that a reasonable number of values fall into each class.

For the convexity maps, zero is the most appropriate central division: by trial and error it was found that further limits of ±3 and ±6 degrees/100m for profile convexity, ±20 and ±40 degrees/100m for plan convexity produce reasonable differentiation for Cache and the 100m-mesh matrices. A one-ended grey scale of the type selected is not ideal for convexity maps; convexities are prominent but concavities are not, unless the symbolisation is reversed. However, it is doubtful if much more can be achieved on the line printer with one symbol per value: the next step might be to produce maps in two colours.

Mean values and ranges of altitude and of gradient vary so widely that it is

necessary to calibrate the class intervals to the data. This is done by setting the central class limit at the mean, thus guaranteeing at least some differentiation, and setting the others at ± 0.6 and ± 1.2 standard deviations. With these class limits, a normal distribution is divided into six classes with 11.5, 15.9, 22.6, 22.6, 15.9 and 11.5 percent of the values (Evans, 1972a, p.105). The outermost classes, covering the two tails of the frequency distribution, are open-ended; otherwise, the class widths are equal at 0.6 standard deviations. Both mean and class widths are rounded; to the nearest metre for altitude, and to the first decimal place for gradient.

A considerable amount of skew or kurtosis can be tolerated before the utility of such limits is appreciably reduced: the worst case here is that of gradients for Cache 2, where the skew of +1.8 gives a standard deviation in excess of the mean. Since negative values are impossible, the -1.2 standard deviation class limit is not brought into play and the map becomes a five-class one (Fig.38). This still shows adequate contrasts, demonstrating the robustness of this class interval system for unimodal distributions without excessive tails.

To permit comparison between areas, a second gradient map is produced, with fixed class intervals in a rounded approximation to a geometric progression (2,5,10,20, 40 degrees). This rarely shows as much contrast as the calibrated map, but usually at least two classes of gradient are well represented. Such a fixed interval map would be pointless for altitude, which varies much more widely.

Aspect cannot properly be represented by a one-ended grey scale; its symbolisation should reflect the circular nature of its scale of measurement, with maximum contrast between opposed aspects, i.e. those 180° apart. Unlike the other properties, aspect lacks tails of extreme values; its range is limited to 360°, and most of this range is present in any but a spatially very restricted terrain sample. Hence there is no need to calibrate class limits for aspect; the same limits can be used for all areas. A set of eight 45° classes is used, centred on the eight principal points of the compass. Symbols were chosen to give maximum contrast between (dark) north - and (light) south - facing slopes: this appeals to verisimilitude in relation to radiation receipts (in the Northern Hemisphere). The transition from north to south can take two routes; via east, or via west. It is necessary, then, to contrast east and west by use of

a second graphic variable. Because of the poverty of symbolisation available for a monochrome line printer map, the second variable must be qualitative. It was decided that circles would be used for slopes with a west-facing component, while crosses were used for those with an east-facing component. With overprinting for aspects from west to northeast, this was implemented by the following symbolisation;

	northeast	east	southeast	
	X+	X	+	
north OXAV				. south
	OX	O-	O	
	northwest	west	southwest	

The results, especially in well-dissected relief, are quite pleasing (Figs. 24 and 37). It must be remembered, however, that aspect means much more on high-gradient slopes than on low. It may be useful to exclude all slopes below a threshold gradient, as on some maps produced by Jasbir Gill (Fig.14). Alternatively, a way of mapping both components of slope (aspect and gradient) simultaneously may be devised. For this it is advisable, finally, to leave the line printer.

The program OYPL produces 'arrow plots' on the graph plotter. Each slope vector is represented by an arrow, centred on the grid point to which it relates (i.e. the centre of the 3 x 3 submatrix). The direction of the arrow gives aspect, which does not require grouping into classes : the plotter used has a .005 inch (.0254mm) increment in any of 16 directions. Gradient is grouped into six classes and, as for the line printer maps, these are either fixed or calibrated. The length of the arrow shaft is graded according to gradient class; the lowest class has no shaft, and succeeding classes have lengths 1/5, 2/5, 3/5, 4/5 and 5/5 of the distance between grid points. The longest arrows can just touch, if aspect is consistent. On the whole, the calibrated plots, with a fairly equal distribution of values between classes of arrow shaft length, show contrast and hence landform more effectively. There is some attraction, however, in class limits which increase the proportion of long arrows, giving an impression of greater continuity along slope lines : these verge on 'flow-line maps' for surface flows of water, though no calculations are performed here to link successive values along a slope line.

Seven 'arrow plots' are reproduced in this report, having been plotted in ink on film with a drum plotter (Figs 6 to 12). Photoreduction to fit on A4 generally improves their appearance; for the originals, there is an optimum viewing distance of about 2m. The Ferro and Cache plots are discussed in the two following chapters. The Torridon, Thvera and Nupur plots highlight the cirque and trough walls. Cirques are apparent especially in Thvera; in Nupur, the upland plateaux are clearly shown, while in Torridon the broad trough floors are marked and the mountains are well separated from each other. Only Thvera is dissected into a network of narrow ridges.

To coordinate with the statistical analysis, maps are produced in this integrated system only as output from a digital computer. As discussed in section (2c) above, altitude matrices form a much better basis for mapping than do surface-specific DTMs, for example, meshes of irregular triangles. Starting from contour maps, however, there are two further alternatives; optical and manual processing.

Optical techniques were pioneered by Gilman (1973; Gardiner and Rhind, 1974) and are now in routine commercial use, at least in the U.S.A. By a two-stage process of contour thickening and thinning, areas where contour spacing is less than a given threshold may be delimited. The process is repeated for each class limit, and printing plates for colour or intensity separation are produced by optical subtraction. Contour plates need to be of high quality with consistent line thicknesses (the thickening of master contours in most map series is unfortunate), and they are pre-edited to remove contour labels, hachures and spot heights. Even so, the results can be misleading on floodplains (with complex, sinuous contours), and blotchy elsewhere. Their main drawback is their inflexibility; if different thresholds become signifcant and a new map is to be produced, the process (except for editing) must be repeated. Also they do not lead on to the production of convexity maps; it is not feasible to optically process an optical slope map in this way since it is essentially a dasymetric rather than a contour map. Statham (1976) discussed the comparable production of orientation (modulo 180 degrees) maps, but these are not aspect maps.

Manual methods of slope mapping have been reviewed briefly by Gardiner (1978) : Krcho (1973) produced maps of all derivatives manually. It is difficult to compare the degree of detail produced by manual and automated techniques : Gill's accuracy check

(section 2d) suggests that there is little difference, but the technique illustrated by Denness and Grainger (1976) seems to take in more of the detail present in a contour map than does grid sampling. The question is, whether particular contour maps are sufficiently accurate to merit such detailed analysis (cf. Clayton, 1953). A small amount of generalisation, as provided by grid sampling, is useful when the contour map is known to have some error.

The disadvantages of all the manual methods of slope mapping are the great amounts of time and effort which they require and the inflexibility of their single end product; to select a new set of class intervals requires repetition of the whole operation. Even the creation of an altitude matrix, onerous though it is (the Thvera matrix required two weeks work), is faster than detailed manual slope mapping: it also permits a variety of maps to be produced, each with known and constant resolution. Where, as discussed in section (2c), matrices are available as photogrammetric by-products without such manual effort, manual slope mapping is totally uncompetitive. Its future appears to be confined to small-scale, one-of-a-kind projects where limited areas are mapped; even there, it is not ideal. For any extensive area, slope mapping should be by computer processing of an altitude matrix. For no extra data cost, this permits production of maps of profile and plan convexity, of aspect, of various combinations of surface properties, and portrayal of the land surface by block diagrams (McCullagh & Sampson,1972; Peucker,1972) and hill shading (Brassel, 1975).

(3e) Within-area relationships

Within each area, relationships between the five basic surface properties are of interest. Since these properties have been defined as relatively independent descriptors, no linear relationships between them are necessarily strong. No universally valid relationships are expected. With the exception of a single non-linear one, relationships between basic surface properties are characteristics specific to each area. By far the strongest and most persistent relationship is the increase of gradient with altitude; even this is only moderately strong, and is sometimes (though not in the present examples) absent or even reversed.

Relationships between surface properties therefore provide further descriptors of each matrix or region, but it is expected that, in any one area, only a few relationships will be of any strength; the remainder are provided for inspection only, as a check. Relationships are expressed in terms of:

(i) product-moment correlation coefficients, except for aspect where periodic regressions on its sine and cosine are used;

(ii) multiple regressions which are linear except for aspect and the inclusion of altitude squared; and

(iii) scatter plots for each of the ten pairs of basic properties.

The latter are indispensible, since many of the relationships are complex and non-linear. However, concise numerical summaries such as correlation coefficients are also useful in most cases; situations where they are misleading are noted below. The construction (a) and interpretation (b) of all three expressions are discussed in turn.

(ia) The summary statistics page (Table 4) gives a matrix of correlations between the four basic properties for which linear relationships might be meaningful, thus providing six different pairs. Zero-gradient points are excluded throughout. As is conventional, values of r (Pearson's product-moment correlation coefficient) are given, so that the positive or negative sense of relationship is portrayed; however, it should be remembered that r^2 is a better measure of the strength of a linear correlation. An r value of .1 (r^2 = .01) is trivial, an r of .3 (r^2 = .09) is very weak, an r of .5 (r^2 = .25) is weak, and an r of .7 (r^2 = .49) is only moderate.

Table 7. Within-area product-moment correlations for the eight matrices analysed to date. Alt = altitude, grad = gradient, profc = profile convexity, planc = plan convexity, alt 2 = altitude <u>and</u> altitude squared. Values for aspect are multiple correlations for cosθ and sinθ.

AREA	alt 2 : grad	alt : grad	alt : profc	alt : planc	alt : aspect	no. of points :
Cache 1	.319	.288	.123	.072	.162	8540
Cache 2	.799	.740	.205	.058	.137	8983
Cache 3	.670	.593	.272	.116	.160	9481
Torridon	.547	.546	.284	.170	.105	9372
Thvera	.149	.119	.346	.206	.103	9993
Nupur	.450	.025	.467	.179	.194	6013
Ferro	.470	.450	.177	.048	.157	11582
Gold Cr.	.599	.173	.155	.098	.493	3447

	grad : profc	grad : planc	grad : aspect	profc : planc	profc : aspect	planc : aspect
Cache 1	+.036	-.001	.074	.085	.056	.023
Cache 2	-.007	+.007	.081	.116	.029	.029
Cache 3	+.006	+.065	.112	.151	.059	.052
Torridon	-.016	+.054	.221	.148	.034	.053
Thvera	-.063	+.016	.184	.174	.034	.049
Nupur	-.054	+.034	.090	.133	.037	.009
Ferro	+.100	+.039	.041	.197	.020	.019
Gold Cr.	-.036	-.072	.126	.159	.106	.089

To obtain corresponding values for aspect (θ), the values of 'multiple R' for regressions on ($\cos\theta$, $\sin\theta$) may be taken and entered on the summary page as a right-hand column in the correlation matrix. (It should be noted, however, that multiple R is never negative). These regressions are obtained from the MIDAS (Michigan Interactive Data Analysis System) package program - which is called by the main program - and they follow the histograms in the computer output. The regression coefficients may be interpreted from the fact that $\cos\theta$ passes through a maximum at 0° (north) and a minimum at 180° (south); it is zero at east and west, positive in the northern semicircle and negative in the southern. Sin θ passes through a maximum at 090° (east) and a minimum at 270° (west); it is zero at north and south, positive in the eastern semicircle and negative in the western.

(ib) However, there are no interesting correlations with aspect within the eight matrices considered here (Table 7); scatter is great in each case. The strongest correlation (r) of aspect with profile (or plan) convexity is a mere .106 for Gold Creek, the strongest with gradient is only .221 for Torridon and, apart from .493 for the very small Gold Creek drainage basin, the strongest correlation with altitude is .194 for Nupur. Inspection of the corresponding scatter plots shows that such relationships as exist are due to the absence of particular aspects at certain altitudes, which usually reflects the way each matrix is delimited: this is rather disappointing. Aspect was expected to have stronger relationships with processes in these steep areas, and thus to affect gradient.

Despite their weakness, relationships between gradient and aspect in glaciated mountains are stronger than elsewhere (Table 8). In Nupur the maximum gradient is a little north of east, corresponding to the gradient-weighted vector mean aspect of 079°. But in Torridon and Thvera, the maximum predicted gradient is just south of west (both sin and cos coefficients are negative). In all three areas, however, scatter plots show that the steepest slopes are north-facing : this illustrates the danger of interpreting very weak relationships.

Of all the bivariate relationships, that between altitude and gradient is strongest. r reaches +.546 in Torridon, and higher values in Cache 3 (+.593) and Cache 2 (+.740): yet, as noted below, this relationship is often non-linear.

Table 8. Periodic and multiple regressions for three matrices in glaciated mountains, in Scotland and Iceland. Bracketed terms contribute a negligible amount to R^2, the coefficient of determination. Values of R^2 are given first for one-variable regressions on the first variable, i.e. altitude, then for the first two variables together, and finally for all four controlling variables.

MATRIX:	DEP.VAR.	CONTROLLING VARIABLES	COEFFS. OF DETERMINATION (R^2) 1-VAR	2-VAR	TOTAL
Torridon:	Grad	$= -.767 \cos\theta - 3.395 \sin\theta + 15.386$	.049		
Thvera:		$= -.703 \cos\theta - 2.701 \sin\theta + 21.562$	.034		
Nupur:		$= +.267 \cos\theta + 1.615 \sin\theta + 21.860$	.008		
Torridon:	Grad	$= +.03339\ \text{alt}\ (+.000005\ \text{alt}^2) -.521 \cos\theta -2.556 \sin\theta -.753$	.298	.299	.326
Thvera:		$= +.03273\ \text{alt} -.000017\ \text{alt}^2 -.536 \cos\theta -2.714 \sin\theta + 6.756$	.014	.022	.056
Nupur:		$= +.12591\ \text{alt} -.000145\ \text{alt}^2 + .416 \cos\theta +1.167 \sin\theta -.108$	.006	.185	.190
Torridon:	Profc	$= +.02313\ \text{alt} -.191\ \text{grad}\ (+.289 \cos\theta) -.491 \sin\theta -8.284$	.081	.122	.124
Thvera:		$= +.01583\ \text{alt} -.107\ \text{grad}\ (+.086 \cos\theta) -.423 \sin\theta -11.738$	.120	.131	.132
Nupur:		$= +\ 0.2984\ \text{alt} -.070\ \text{grad} +.520 \cos\theta +1.156 \sin\theta - 11.539$	.218	.222	.228
Torridon:	Planc	$= +.07932\ \text{alt} -.351\ \text{grad}\ (+1.11 \cos\theta) -4.18 \sin\theta -27.53$	.029	.031	.033
Thvera:		$= +.05375\ \text{alt} -.088\ \text{grad}\ (-.43 \cos\theta) -3.16 \sin\theta -44.86$	.042	.044	.044
Nupur:		$= +.05043\ \text{alt} +.116\ \text{grad} +1.95 \cos\theta\ (+1.07 \sin\theta) - 26.09$	.032	.033	.033

Linear relationships between gradient and both convexities are negligible, but there is a very weak tendency (.174 > r > .085) for plan and profile convexity to increase together (Table 7). Both convexities increase with altitude, but profile convexity always has the stronger correlations. Both correlations are stronger in the three glaciated mountain areas than anywhere else, but are still weak : in these three areas, correlations between profile convexity and altitude range from .284 to .467, while those between plan convexity and altitude range from .170 to .206. One might expect that the convexities of ridges would generally occur at higher altitudes than the concavities of valleys, but in practice these relationships are very weak.

(iia) A limited number of multiple regressions are included in the output; these are not 'shotgun' regressions, but deal only with relationships which might be causally meaningful. Hence altitude and aspect are treated as controlling ('independent') variables; their effects on climate and hence surface processes may influence the other surface properties. Gradient is seen as a function of these two, while profile and plan convexity are seen as functions of altitude, gradient and aspect. Because of the curvilinearity apparent in some scatter plots (Fig.42), a term in altitude squared is included in the regression for gradient only: each regression, then, has four controlling variables.

These regressions, too, are performed by the MIDAS package. In the latest version of the terrain analysis program, stepwise regression is requested and each step is printed out. However, to facilitate comparison of coefficients, the full four-variable regressions are given in Table 8. Deciding which variables to omit from a multiple regression is always a difficult matter, and here it is even more so because with a 'sample' of some 10,000 values almost any relationship is naïvely 'significant'. The 10,000 values, however, are not independent; their autocorrelation biases significance tests. In the circumstances, it is best to avoid the matter of statistical significance and judge each variable in terms of its effect on R square : variables which make no impact on the third decimal place are of no interest, whatever their statistical significance. Those with F values below 1.0 are obviously unimportant, but F should preferably be well above 1.0.

(iib) Addition of a term in altitude squared increases correlation of gradient with altitude in all areas except Torridon : the most remarkable increases are from 0.25 to .430 for Nupur and .173 to .599 for Gold Creek. This shows that the most common relationship (in this limited sample of areas) is parabolic (Fig.42); in general, gradient increases with altitude, but this is reversed at high altitudes by gentle slopes on ridge tops, especially summit plateaux such as those of Nupur. The quadratic term adds much less in Thvera and Torridon, where summit ridges are sharper and there are only traces of high plateaux.

Further extension of the regressions to include aspect results in improvements of a few percent in R^2, but for Thvera even this change represents more than a doubling of the correlation strength. The signs of the aspect regression coefficients are identical to those for periodic regressions on aspect alone, and the magnitudes are only slightly smaller (Table 8); the additional R^2 equals that for aspect alone for Thvera, and is 60% of R^2 for aspect alone for Torridon and Nupur. Hence the effect of aspect does not overlap excessively with that of altitude.

Multiple regressions for the convexities show that gradient and aspect add little to R^2, once altitude has been taken into account. Coefficients for altitude remain positive, while those for gradient are negative except for plan convexity in Nupur. Only in Torridon does gradient contribute notably (to profile convexity), and this is not predictable from the simple correlation of only -.016. In no case does aspect increase R^2 by more than 1% of total variance in convexity.

(iiia) The scatter plots which illustrate the pairwise relationships (Figs.28-30, Figs.41 & 42) are produced by SPSS. On the line printer the plots are 50 printer units high and 100 wide : hence the horizontal scale is divided to twice the resolution of the vertical. By calling each plot separately, it is possible to determine which variable plots on which axis; the first-named is on the vertical axis. It is conventional to plot controlling variables on the horizontal axis, and dependent variables on the vertical. By specifying the order (profc, planc, gradient, altitude, aspect), profile convexity is always on the vertical axis (dependent), while aspect is on the horizontal (controlling). Gradient is on the horizontal axis when plotted against either convexity, but on the vertical when plotted against altitude or aspect.

A common problem with computer-produced scatter plots is that the default procedure for scaling is equal division of the range, which gives scale values with several decimal places. The options used here round altitude, gradient and aspect to the nearest whole number (metres or degrees) for plotting the scale values, and specify the maximum and minimum values of profile convexity as ± 50 degrees, and of plan convexity as ± 500 degrees, giving highly rounded values. Scale values are printed for both extremes and for ten intervening values, on both axes. (Unfortunately, it is not possible to suppress the two decimal places in scale values, even though these are always zero).

The number of values excluded from the plot (by setting maximum and minimum values for the convexities) is printed beneath; obviously the plots should be re-run without the range constraint if the number of excluded values is large. The SPSS program also gives details of the linear regression for plotted values; although this is usually of little interest, it does provide a check on the effect of outliers on linear correlation coefficients.

The scatter plots contain a star for every printing position in which a single point falls. If more than one point falls in the printing position, the corresponding digit (2...9) is printed. The digit 9 is used also for all frequencies greater than 9. With some 10,000 points and 100x 50 printing positions, much of each plot consists of digits, and 9 is very common; the program was designed essentially for smaller data sets, and frequent use of digits does not provide a clear graphic design. It would be useful if a new scatter plot program were written, using overprinting to provide a grey scale effect for the density of points. However, the present program does provide a quick general impression of the scatter of points.

(iiib) Apart from the relationships noted above, two further ones are apparent from the scatter plots. Although plan convexity is almost uncorrelated with gradient, its modulus declines with increasing gradient, i.e. on steep gradients, contours are relatively straight. The variability of contour curvature increases steadily on gentler slopes, where both greater convexities and greater concavities are found (Fig.30). This relationship is found in all eight areas. It does not apply to profile convexity, which is equally variable on all gradients. (As changes in altitude are reduced in most

areas of low altitude, one might in fact expect changes in gradient to be reduced in areas of low gradient, but this is not the case). The dominant factor seems to be that changes in aspect are 'easier' where gradient is low ; a small displacement of the slope vector can give a large change in aspect.

The second relationship is that, while profile convexity increases (slightly) with plan convexity, the plot is in fact cross-shaped (Fig.29). The positive correlation comes from skewing of the arms of the cross. Finally, it should be noted that the relationship between gradient and altitude is usually more complex than quadratic, and a scatter plot is essential to illustrate this (Figs.28,42).

(3f) Between-area relationships

One use of these statistics and correlations which describe an area is to assess variation between areas. This also provides further insight on the meaning of the statistics. With results (Tables 5 and 7) only for these eight matrices however, it is difficult to make confident statements about between-area relationships. Some comparisons have been drawn above between the three glaciated mountain areas, each of which has a 100m mesh. Though the map quality and vertical resolution of encoding (1m) was better for Torridon, these three data sets and that for Ferro are approximately comparable. The Cache matrices have one-quarter this grid mesh, while Gold Creek has a much finer resolution still and cannot really be compared with the other areas.

There do seem to be positive relationships between (i) standard deviation of altitude; (ii) mean gradient; and (iii) standard deviation of gradient. All of these are indicators of 'relief' and surface roughness. The measures of distribution shape - skewness and kurtosis - seem to be related neither to any of these, nor to each other except for the tendency of kurtosis to increase with the modulus of skewness.

For profile and plan convexity, standard deviations are unrelated to the other measures and to each other. Because convexity distributions are expected to be symmetrical about zero, increasingly positive mean values should relate to increasingly negative skewness, and vice versa. This expectation does not materialise, due probably to the greater effect of extreme values on skewness. Their even greater effect on kurtosis reduces the value of that statistic, and makes comparison of convexity kurtosis between data from different sources impossible.

No relationships are apparent in the way correlations vary between areas. Why, for example, should the altitude : gradient correlation be much weaker in Thvera than elsewhere? The low correlation in Cache 1 reflects generally low relief, but this does not apply to Thvera. The only other correlation to produce reasonably strong values is that between altitude and profile convexity, and again no patterns are apparent.

It seems, then, that these statistics (and correlations) are relatively independent descriptors. Although the three relief variables are generally intercorrelated,

differences between them will reflect special types of topography. A field of sand dunes, or of thermokarst 'cemetery mounds', could have steep gradients without much (regional) variation in altitude. A dissected plateau, a glaciated area and a field of inselbergs would have higher variation in gradient than a well-integrated fluvially-dissected area of the same mean gradient.

It is necessary to accumulate further evidence on these properties, but the descriptors chosen do seem to be useful and distinct.

(3g) The importance of grid mesh

A previous Report (Number 3) was devoted to the effect of grid mesh (resolution) on gradients. A broad range of resolutions was investigated by thinning out the original data matrices; from 25 to 625m for Cache, and from 100 to 2500m for Torridon. Although the results for the two study areas were somewhat different, an exponential decline of gradient with increasing grid mesh was suggested; a power function was a second possibility. Standard deviations were more sensitive to grid mesh, that is, they declined more rapidly than means as mesh increased. Schloss (1966) used a power function for the decline in curvature between 0.4 and 12.5m mesh. We are more concerned with the finer end of the scale, since each of these data sets is coarser than the resolution at which geomorphologists interested in surface processes prefer to calculate gradient. Here, variations are appreciable but not drastic. For Torridon, mean gradients of 12.0 degrees at 400m, 13.7 at 200m and 14.8 at 100m may be extrapolated to about 15.7 degrees at 1.5m, a distance over which gradients are commonly measured in the field. For Cache 3, mean gradients of 4.9 degrees at 100m, 5.5 at 50m and 5.9 at 25m may be extrapolated to 6.1 degrees at 1.5m. Differences in results between Report 3 and Table 5 are due to inclusion of zero gradients in the former, rather than to the slightly different algorithm used for calculating gradient.

Corresponding results are not available for the other statistics, but Table 9 provides a comparison of 100m, 200m and 300m meshes for the central part of the Nupur matrix, as outlined on Fig.13. Not being based on derivatives, altitude statistics are almost unaffected by grid mesh. Differences in maximum and minimum are due to loss of peripheral rows and columns in coarser matrices. As for Cache and Torridon, mean gradient diminishes slowly with increasing mesh and standard deviation of gradient declines more rapidly. We might extrapolate to a mean gradient of 23.1 degrees for 1.5m mesh. Since maximum gradient is reduced more rapidly than mean, the (slight) positive skewness is less for meshes of 200 and 300m than for 100m. Kurtosis becomes somewhat less extreme. Maps of gradient at each grid mesh (Fig.15) show how the thin belts of steep gradients are generalised out at the coarser resolutions. With a 300m grid mesh, only the longest steep slopes retain gradients over 36 degrees;

Table 9 The effect of grid mesh on statistics for the central half of the Nupur matrix. All possible submatrices were included for each mesh, i.e. the 100m data were used as fully as possible. Figures are rounded to approximately 3 significant digits, and were supplied by J.S. Gill.

ALTITUDE

MESH	MEAN	SD	SKEW	KURT	MAX	MIN
100m	471	196.2	-.367	-.88	776	29
200m	475	192.6	-.369	-.88	786	34
300m	482	185.4	-.365	-.89	806	56

GRADIENT

MESH	MEAN	SD	SKEW	KURT	MAX	MIN
100m	21.13	12.13	+.178	-.97	55.32	0.00
200m	19.36	9.56	+.102	-.81	44.05	0.00
300m	17.23	7.71	+.114	-.58	38.82	0.45

PROFC

MESH	MEAN	SD	SKEW	KURT	MAX	MIN
100m	.635	11.21	+1.556	+5.90	+100.6	-42.2
200m	.619	6.84	+ .836	+ .98	+ 38.8	-21.2
300m	.634	4.78	+ .503	+ .04	+ 20.1	-10.4

PLANC

MESH	MEAN	SD	SKEW	KURT	MAX	MIN
100m	-.023	52.7	+3.70	+80	+999	-688
200m	1.937	98.4	+45.46	+2322	+5042	-359
300m	.551	37.6	-3.48	+127	+418	-823

CORRELATIONS

	ALT : PROFC	ALT : PLANC	ALT : ASPECT(linear)
100m	.459	.171	.212
200m	.605	.102	.180
300m	.684	.262	.168

the others become wider belts of moderate gradient. The basic spatial pattern, however, remains the same. Note that in these maps gradients below 6 degrees are left blank.

Fig.14 shows aspect, generalised in a slightly different manner. The top map is based on the original 100m data : the middle one (T2) on altitude data smoothed by the weighted filter 1-2-1 in both directions, and the bottom one (T3) on altitude data smoothed twice by this filter. The smoothing produces no drastic changes, but the map is simplified so that there are larger blocks of the same aspect class. Gradients below 20 degrees are excluded, so that the maps concentrate on those where aspect assumes greater significance: thus the area covered is also simplified in outline from T1 to T3. Three maps of gradient based on the same smoothed data are very similar to those of Fig.15.

As expected, the second derivatives are more sensitive to grid mesh. The magnitudes of extrema of profile convexity are drastically reduced with increasing mesh, as is the standard deviation; skewness and kurtosis rapidly approach their Gaussian values. Fig.16 shows how edges are blurred by increased mesh. Convexities remain sharper than concavities ; about twice as sharp, for the extrema.

Moment statistics of plan convexity behave erratically as grid mesh varies, providing further confirmation of their instability. The 200m results are probably influenced unduly by an extreme convex value of 5042 degrees/100m, giving high dispersion and a ridiculously positive skew. This does not prevent the maps of plan convexity from providing a consistent and interesting impression (Fig.17). The simpler 300m - mesh view picks out the major ridges and valleys, which are confounded with details at 100m. The 200m-mesh view is more similar to the 300m than to the 100m. In terms of maps, then, variation of grid mesh is constructive and helpful for plan convexity, whereas for the other derivatives the related generalisation is less necessary and less interesting. Note that the scale for Figs.16 and 17 differs from that for Figs. 26, 27, 39 and 40; use of white for extreme concavity makes valleys much more apparent in Fig.17.

The moderately strong correlation of profile convexity with altitude increases with grid mesh. The linear correlation of aspect with altitude is somewhat reduced, and that of plan convexity with altitude is variable. These results are presumed to be specific to the area considered.

The effect of grid mesh varies, then, between the geomorphometric statistics proposed here. The incidence of differences (Table 9) is unlike that between adjacent areas (Table 6a). Altitude is unaffected, save for sampling variation. So long as a reasonable body of data is incorporated, altitude statistics based on samples differing in mesh may be compared for areas which are in some way comparable. For gradient and for correlations, such comparison would be subject to considerable error. For profile and plan convexity, any comparison based on data differing in grid mesh would be vain: furthermore, the data should be generated in comparable fashion.

Chapter Four

DETAILED EXAMPLE OF ANALYSIS OF A DRAINAGE BASIN : FERRO, N. CALABRIA, ITALY.

(4a) Introduction

The terrain analysis program deals essentially with areas rectangular in outline, but irregular areas can be analysed by inputting altitudes for the whole of the containing rectangle, leaving blank points which fall beyond the outline. These are treated as missing values and excluded from all calculations and tabulations. A quadratic is fitted only when all nine altitudes are non-zero. It is much more convenient to leave external areas blank (read as zero) than to punch a number such as -999 in every position. If coastal areas with zero or negative altitudes are to be processed it will be necessary either to change the program or, more simply, to pre-edit the file by adding a constant to each altitude.

The Fiume Ferro is a short river in northernmost Calabria, Italy. It rises at around 1,000m, south of Oriolo but north of Monte Pollino, and flows some 32km to reach the Gulf of Taranto near Amendolara, near the middle of the 'instep' of Italy. Its steep course is eroded into recently uplifted Cenozoic clays and sands. Gullying and landslides are common, and in connection with a survey of these problems (Carrara, Carratelli and Merenda, 1977) an altitude matrix was generated. Altitudes were encoded in 10m units for a 100m mesh matrix, from 1/10,000 maps with a 10m contour interval. Although interpolation of altitude to 2m might have been justified, the coarser encoding speeded data capture and was adequate for the high relief involved. The grid mesh in any case generalises out minor gullies, but represents the slopes into which they are incised. The matrix is contained within a 242 x 128 rectangle, but this provides only 11,796 3 x 3 neighbourhoods for quadratics. The region is sharply dissected and without summit plateaux : the only extensive level areas are the floodplains of the Ferro and its main tributaries, which are braided and choked with sediment. These provide all 211 plain and 3 pit points : there are no other zero gradients.

(4b) Histograms

Altitude ranges from 7 to 1148m, with a mean of 445m, median of 431m and mode of about 438m. It has a weak positive skew, mainly because the concave-up tail on the high side is not balanced by one on the low side (Fig.18). Inclusion of zero-gradient points reduces the mean by 6m, but increases the standard deviation.

The histogram of aspect (Fig.19) is fairly even except for a deficit of west-facing slopes : this is because the trunk stream flows first northeast, then southeast. The resultant vector is somewhat north of east: since many of the eastward slopes are on valley-floors, the strength of the resultant is reduced from 12.6% for unit vectors to 2.9% for gradient-weighted vectors.

Gradients are unskewed, being tightly clustered (S.D.=5°) around their 13° mean. 13° is also the median, though the mode is shared between 11° and 13° with 17% fewer observations at 12°. There is a slight positive tail up to a maximum of 35°, but this is balanced by relatively high frequencies below 7°.(Fig.20). When low gradients are plotted in .1 degree classes, many classes are empty. This is due to the coarse (10m) vertical unit used; the problem abates with increasing gradient.

Profile convexity has a very symmetrical, long-tailed frequency distribution (Fig.21), but in the range 0.5 to 20 degrees/100m there are 25% more concavities than convexities. Hence both mean (-.61) and median (-.66) are displaced below zero. Although there are 21 convexities in the 30-40 degrees/100m range compared with 10 concavities (Table 10) this does not balance the bias toward gentle concavities : presumably some net concavity is due to closure of the drainage basin. The kurtosis of over 11 reflects the length of both tails compared with the breadth of the histogram peak. An early run contained a data error where 71 (tens of m) had been transposed to 17. This was detected in terms of a 'pit' in the arrow map and eight positive outliers of gradient and profile convexity, compared with only one negative outlier of the latter. The effect of this imbalance was to change skewness of profile convexity from +.057 to -.946, and the presence of outliers increased kurtosis from 11.7 to 52.6. This was a salutary reminder of the effect of a single large data error on higher moments of derivatives.

Table 10. Summarised frequency distributions of convexity for Ferro, Italy. Numbers of convexities (+) and concavities (-) are juxtaposed, for each magnitude range (degrees per 100m in each case).

PROFILE CONVEXITY

−		+
3	above 50.0	2
3	40.5 to 50.0	4
10	30.5 to 40.5	21
57	20.5 to 30.5	55
451	10.5 to 20.5	315
4479	1.5 to 10.5	3485
859	0.5 to 1.5	803
5862	totals	4685
+0.5 to −0.5	1035	
overall total	11582	
median	−.583	
skew	+.057	

PLAN CONVEXITY

−		+
11	above 500	12
7	405 to 500	3
21	305 to 405	17
42	205 5o 305	36
230	105 to 205	183
3368	15 to 105	3366
1299	5 to 15	1352
4978	totals	4969
+5 to −5	1635	
overall total	11582	
median	−.003	
skew	−1.521	

Plan convexity has even longer tails (compared to its peak), and a sharp mode at zero which is also the median (Fig.22). The mean at −.81 is only slightly negative, compared with the standard deviation of 71, but there is a negative skew of 1.5. A slight excess of weak convexities is balanced by a surplus of moderate concavities, but the tails are balanced.

(4c) Maps

The map of altitude shows the basin divided into a broad upper, northeast-facing valley and a narrow lower, southeast-facing valley which is continued by the Canale Raia tributary around Oriolo. The line printer map is printed in two strips, of which only the larger (upper) one is reproduced here (Fig.23): the full extent of the basin is shown by the arrow plot (Fig.12). The pattern of aspect (Fig.24) is much clearer than for Cache, and shows the integration of this fluvially-controlled topography. Each valley-side has a consistent aspect, and this changes sharply at ridges and streams. The aspect map could pass for a hill-shaded map, illuminated from the south at a high angle.

Gradients are steepest in the upper part of the basin, especially near the concavity in its outline (Fig.25), and also in a tributary basin north of the lower course. Most gradients are between 10 and 20 degrees. The arrow plot (Fig.12) in particular enhances contrasts in gradient (by the mean and standard deviation calibration procedure) and provides a very graphic view of the basin's land form. More than the other areas analysed, this basin approximates to Strahler's (1950) conditions of uniformity of lithology, soils, vegetation, climate and stage of development, so that gradients are almost normally distributed, with low variance. (Strahler's original statement applied to maximum slope angles of valley sides. In this case it can be extended to the distribution of all slopes).

In the map of profile convexity (Fig.26), the most consistent pattern is of concavity, several mesh units wide, along the valleys. (This would be more apparent if the scale of shadings was reversed). It is interrupted on the flood-plain of the mainstream by some gentle convexities, due perhaps to rounding of the data to 10m. The pattern of sharp convexities along ridges is less continuous. In the tributary basin noted above for its steep gradients, it is the lower slopes which are convex.

Plan convexity captures the dendritic ridge line pattern more coherently (Fig.27). Ridge lines often continue down to valley floors, just as valley lines continue up to cols. Strong plan, but not profile, convexities are possible on gentle slopes.

(4d) Relationships

The strongest relationships between the local variables are given in Table 11. Visually, the increase in gradient with altitude (Fig.28) is more linear than is often the case; the quadratic term is relatively ineffective over the 1140m range in altitude and adds only .019 to r. Aspect makes negligible contributions to other regressions. The increase of profile convexity with altitude (r = .183) is very slight and the scatter plot is no more impressive than that of plan convexity against altitude (r = .050): outlying points seem to have the major influence.

Table 11. Multiple regressions for Ferro, Italy. Negligible contributions are omitted. Altitude is expressed as a function of aspect (θ); gradient and profile convexity as functions of altitude; and profile convexity as a function of plan convexity.

Alt = + 44.6 sin θ + 444.3	R = .157	R^2 = .025
Grad = + .02188 alt − .0000114 alt^2 + 6.090	R = .470	R^2 = .221
Profc = + .00594 alt (− .0226 grad) − 2.95	R = .183	R^2 = .033
Profc = + .02486 planc − .59	r = .205	r^2 = .042

(28 extreme values excluded from the latter)

At any aspect, a broad band of altitudes are present, but the highest part of the basin has east-and north-facing slopes. The result is that the periodic regression of altitude on aspect is a positive function of sin θ, with the cos θ term negligible, so that the regression passes through a maximum (489m) at east (090°). Despite the lack of west-facing slopes at the lowest altitudes (where the mainstream flows east-southeast), minimum altitude (400m) is predicted at west. The amplitude of the regression is considerably less than one drawn by visual fit.

The relationship between profile and plan convexity (Fig.29) (r= .170) here is one of the strongest observed, and is increased to r = .205 when values more extreme than $\pm$ 505 degrees/100m in plan convexity, or $\pm$ 50.5 in profile convexity, are excluded. The scatter plot suggests a slightly skewed cross, with

variations of each variable greatest when the other is near-zero. The slight tilting of each of these belts is responsible for the positive correlation.

The plot of plan convexity against gradient (Fig.30) is a salutary reminder that lack of correlation does not imply lack of relationship. The scatter of points is triangular, centred on a plan convexity of zero and with a broad base. Below 8° gradient, plan convexities range to over $\pm$ 500 degrees/100m. Between 8° and 17° they are below $\pm$ 400, and mainly below $\pm$ 150. Above 25°, they are all below $\pm$ 100. Such a relationship is typical of this pair of variables.

Finally, the plot of gradient against aspect (not reproduced here) shows that only a limited number of combinations is possible. This effect is due to rounding of altitude to 10m, and is most marked on gentle gradients and near the axial directions. It has little consequence for the statistical descriptors.

The Ferro basin, in summary, is a well-dissected area of moderate and steep slopes, with relatively little variation in gradient. Both crests and valleys are sharp, and there is a broad range of altitude. Frequency distributions of altitude and gradient are almost unskewed. Fuller interpretation will be possible when results from a number of fluvial basins are available.

Chapter Five

DETAILED EXAMPLE OF ANALYSIS OF A RECTANGULAR AREA: CACHE, OKLAHOMA, U.S.A.

(5a) Introduction

The Cache area lies at the foot of the Wichita Mountains, and consists largely of the adjacent undulating lowland. It had been chosen for experimentation with altitude matrix data produced by the UNAMACE automated stereoplotter, and related digital mappping. For the present project, data for a 2.5 x 7.5 km area east of Cache, from 41.5 to 44km east and 30.5 to 38 km north on the 1/24,000 map, were made available. Every second point along each traverse was selected, so that a grid mesh of 25m applied along both axes. The area was divided into three 100 x 100 square matrices, which were then analysed separately.

Cache 1, at the southern end, is mainly lowland; Cache 3 at the northern end is upland with broad valleys; and Cache 2 straddles the fairly sharp boundary between the two. Altitudes are much more tightly grouped in Cache 1, where they range from 367 to 398m, than in the other matrices : Cache 2 (Fig.31) ranges from 379 to 483m and has a standard deviation almost as high (Table 5) as Cache 3, which ranges from 403 to 526m. Because Cache 2 contains more lowland than upland, it has a greater (positive) skewness of altitude.

Gradients also increase from Cache 1 to Cache 3, with Cache 2 being precisely intermediate in mean. The mixture of terrains in Cache 2 gives it a gradient variability (Fig.33) greater even than Cache 3. Skewness is positive in all three matrices and increases strongly as mean gradient falls toward the lower bound of zero. This skewness induces a trend to positive kurtosis.

(5b) Histograms

Profile and plan convexity have both positive and negative values (Figs.34,35), and mean values are close to zero as expected. Their standard deviations, which measure the magnitude of curvature whether convex or concave, show little difference between the three matrices. Those for plan convexity are much greater than for non-Cache matrices. The positive values of kurtosis affirm the long tails at both extremes.

Table 12 summarises the frequency distributions of profile and plan convexity. There is a prevalence of profile concavities over profile convexities, but this comes essentially from those gentler than 20°/100m. Really sharp profile convexities and concavities are limited to Cache 1. Although means are negative (concave) in each matrix, interpolated medians are approximately equal to means and the distributions are unskewed except for Cache 1. The strong positive skew of Cache 1 appears to reflect the extreme values, with 13 convexities between 50 and 120°/100m compared with only 6 concavities.

Plan convexities on the other hand are dominantly positive, especially between 15 and 300 $^{\circ}$/100m. Also there are rather more extreme convexities than concavities. Because of these tails, means are more convex than medians, and skewness is somewhat positive. The unusually strong positive skew of plan convexity for Cache 3 is not apparent in the histogram, and may reflect the maximum value of +6,417 compared with the minimum of -1,833. Six convexities, but no concavities, are sharper than 2000 degrees/100m.

All the convexity histograms, especially for Cache 1, are 'spiky', that is, certain classes are much more frequent than adjacent ones. Aspect is similarly affected, with greater frequencies for classes covering the eight most cardinal points (e.g. 225°, 315°) and to some extent, the next eight (e.g. 112.5°, 247.5°) (Fig.32). Such spikiness, or extreme multi-modality, is not a reflection of true surface properties but an artefact of the data. In this case, it seems that the data are inadequate in lowland and valley areas because altitude is recorded to the nearest metre, which is comparable in magnitude to the local relief. Hence adjacent altitudes commonly differ by only one unit, or not at all, and their discreteness limits the number of possible aspects, gradients

Table 12. Summarised frequency distributions of convexity for the three Cache matrices. Numbers of convexities (+) and concavities (-) are juxtaposed, for each magnitude range (degrees per 100m in each case), and zero-gradient points are classified below.

PROFILE CONVEXITY

	Cache 1		Cache 2		Cache 3	
	-	+	-	+	-	+
above 50.0	6	13	0	0	0	0
40.5 to 50.0	3	1	0	0	3	5
30.5 to 40.5	4	14	13	9	17	18
20.5 to 30.5	16	13	33	46	122	120
10.5 to 20.5	127	111	320	275	764	631
1.5 to 10.5	3717	3442	3787	3266	3515	2843
0.5 to 1.5	94	112	168	160	360	342
totals	3967	3706	4321	3756	4781	3959
+0.5 to -0.5	867		906		741	
overall totals	8540		8983		9481	
median	-.15		-.31		-.61	
mean	-.15		-.36		-.60	
skew	+.74		+.08		+.17	

PLAN CONVEXITY

	Cache 1		Cache 2		Cache 3	
	-	+	-	+	-	+
above 500	194	223	110	120	75	98
405 to 500	221	198	157	127	63	65
305 to 405	197	188	130	128	118	106
205 to 305	118	157	57	115	159	138
105 to 205	1513	1630	1360	1485	854	913
15 to 105	987	1069	1505	1705	2202	2858
5 to 15	115	105	253	219	420	408
totals	3345	3570	3572	3899	3891	4586
+5 to -5	1625		1512		1004	
overall totals	8540		8983		9481	
median	+.7		+1.1		+3.5	
mean	+7.2		+4.7		+8.6	
skew	+.48		+.32		+6.80	

ZERO - GRADIENT POINTS

	Cache 1	Cache 2	Cache 3
plains	1011	595	110
summits	9	6	3
ridges	15	4	3
saddles	5	6	3
valleys	13	6	1
pits	11	4	3

and convexities, where gradients are low.

Confirmation of this discreteness in the data comes from the large number of zero-gradient points. In particular, the number of 'plains' (3 x 3 neighbourhoods with all nine altitudes identical) increases from 110 in Cache 3 to 595 in Cache 2 and a remarkable 1,011 (out of 9,604 neighbourhoods) in Cache 1. The (much smaller) numbers of other types of zero-gradient point increase in the same direction (Table 12). Only one zero-gradient point, a summit, lies within the hill area, or above 424m : this shows that the one-metre vertical unit is satisfactory for hill areas though problematic for lowlands. (For the Icelandic mountains, a 5m unit produces very few zero gradients).

Histograms for altitude show that it is far from normally distributed, nor is it simply positively skewed. A tendency to broad peaks produces negative kurtosis except in Cache 2, where the extremely long, flat tail from the small mountain areas produces strong positive skew accompanied by positive kurtosis. (The main part of the distribution is between 379 and 409m, but the tail extends to 483m giving a strong impression of a mixed distribution: Fig. 31). The histogram for Cache 3 is even stranger : its main bulk spans the whole altitude range of Cache 2, but there is a thin higher tail and an important peak at lower altitudes corresponding to a broad intermontane valley. Clearly it is unreasonable to expect altitude to be normally distributed for square areas of this size.

The gradient histogram for Cache 1 has a simple positive skew, but that for Cache 2 (Fig.33) is bimodal with a sharp peak around 1° (as in Cache 1) and a broad peak around 10°. In Cache 3 the latter is broader and more continuous with a peak at 2°, where frequencies are somewhat greater. The maximum is no greater than for Cache 2, so the higher mean gradient (5.8°) comes from the larger proportion of gradients around 10°.

Aspect (Fig.32) shows a tendency to 242° in Cache 1 and 2 and 329° in Cache 3. Due to the way these were formatted, it is necessary to subtract 90° from these figures, i.e. the tendency is really to the south east, or southwest in Cache 3. The tendencies simply reflect the small extent (2.5 x 2.5km) of the areas covered, having only a few major topographic features.

(5c) Maps

Maps of altitude show that the main mountains are in the northeast of Cache 3 and northwest of Cache 2 (Fig.36). In Cache 1 the relatively highest ground is along the western edge, with a small hill also on the eastern edge. Aspect varies rather abruptly in Cache 1, forming a mosaic of small blocks. More consistent patterns are found in Cache 3 and the northwest of Cache 2 (Fig.37).

The basic spatial pattern of gradient (e.g. Fig.38) follows that of altitude, but the hill crests are broad enough to have zones of reduced gradient up to 12 mesh units wide (i.e. up to 300m). This suggests broad upper convexities. The production of a further gradient map with a fixed 'universal' series of class limits (at 2,5,10,20 and 40 degrees) permits the three matrices to be compared, but these maps lose much differentiation and serve mainly to confirm the value of calibrated class limits. The arrow plots (Figs 6 to 8) show the pattern within each matrix, especially the long, flowing slopes of Cache 3, but they should not be compared because of their different class intervals. Use of the same intervals for all three makes Cache 1 a very flat map. Even so, a certain 'blockiness' is apparent in Cache 1 (Fig.6), which presages the problems discussed below.

As higher derivatives are taken, the results are more sensitive to data quality. Maps of altitude, gradient and aspect are perfectly acceptable, but those of profile convexity provide a shock (Fig.39). The lowland areas of all three matrices form a pattern of juxtaposed convexity and concavity which would be chaotic were it not for a distinct east-west lineation, across the scan-lines of the UNAMACE. Sometimes a column of seven values in the top convexity class are juxtaposed with seven in the top concavity class, implying very linear corrugation of the surface at high frequencies. The effect is consistently concentrated on south-facing slopes and at 388-392m altitude.

This lineation cannot be due to the alternation of scanning directions, since that would produce an orientation 90° different : it appears that CONPLOT 2 copes with this problem. Following conversations with Dr. V. Lagarde, lineation is attributed to mechanical hysteresis in the UNAMACE system. It appears to overcorrect

so that errors of alternate sign occur 25m apart along a traverse. This, and the concentration of error on south-facing slopes, may warrant further investigation by those processing UNAMACE data. It also seems that analysis of the present type is useful in testing data quality.

Plan convexity exhibits a blockier spatial pattern, with a certain amount of lineation (Fig.40). Although this has the same orientation as for profile curvature, it is differently located, especially on slopes facing east or west.

There are, however, substantive results from the maps of convexity, within upland areas. The broad convexity of ridges, noted above, is apparent in profile convexity maps, and changes rapidly into the concavity of footslopes. The hill slopes are therefore convexo-concave, with little rectilinear element. Plan convexity maps pick out the pattern of subsidiary ridges well, but convexities are still broad except at cols, where the concavities of valleys on either side often meet. A map with more extreme class limits is more successful in picking out ridge lines and valley lines.

(5d) Relationships

Correlation coefficients below .12 show that linear (or periodic) relations between gradient, aspect and the convexities are weak. This is confirmed by scatter plots, except that plan convexity is more extreme on low gradients, giving a strong triangular relationship. Plan and profile convexity have very weak positive intercorrelations: scatter plots of one against the other suggest four tails forming a slightly skewed cross.

In upland areas, altitude correlates positively but weakly with plan and (especially) profile convexity. Fig.41 shows how, in Cache 2, many more extreme plan convexities and concavities are found on the lowlands below 410m. This is consistent with the contrast, already noted, between Cache 3 and Cache 1. The correlations of .14 to .16 between altitude and aspect (expressed as cos θ, sin θ) simply reflect the limited area covered by each matrix.

By far the strongest correlations are between gradient and altitude, weak only in Cache 1 where the range of altitude is highly restricted. This is not, however, a linear relationship. Both Fig.42 and the other two scatter plots show parabolic relationships, with lower gradients at both high and low altitudes, and a broad maximum between. (Gradients over 15° are found between 415 and 460m in Cache 2, between 422 and 474m and 485 and 506m in Cache 3). Hence inclusion of a term in $(\text{altitude})^2$ increases correlation by some .06. The regression coefficients show that the trend does not reverse within the range of gradients encountered, i.e. gradient goes on increasing with altitude and the equation does not provide a close fit to the observed trends. The complexity of Fig.42 comes from the presence of two rounded ridges, at different altitudes: each produces its own 'limb' of low gradients. It is complexities such as this, and triangular relationships, which make it vital to inspect scatter plots before interpreting even quite high correlation coefficients (.74 in this case for the linear relationship).

Table 13 Multiple regressions for the three Cache matrices. Negligible contributions from aspect terms are omitted. Gradient is expressed as a function of altitude, its square, and aspect. Profile and plan convexity are expressed as functions of altitude, gradient and aspect. Coefficients of determination(R^2) are given for one controlling variable (the first, altitude), for the first two together, and for the complete equation. Note that due to the input format, aspect (θ) has been displaced clockwise by 90°.

MATRIX:	DEP.VAR.	CONTROLLING VARIABLES	COEFFS.OF DETERMINATION(R^2) 1-VAR.	2-VAR.	TOTAL
1	Grad	$= -2.875\ \text{alt} + .0038\ \text{alt}^2 - .105\cos\theta + 538.47$	.083	.102	.107
2		$= +1.904\ \text{alt} - .0021\ \text{alt}^2 - .205\sin\theta - 422.59$	.548	.638	.640
3		$= +1.648\ \text{alt} - .0017\ \text{alt}^2 + .481\cos\theta - 379.73$	.352	.449	.456
1	Profc	$= +.1696\ \text{alt}\ (+.010\ \text{grad}) + .734\sin\theta - 64.46$	.015	.015	.021
2		$= +.1389\ \text{alt} - .617\ \text{grad} - 54.31$	.038	.090	.090
3		$= +.1441\ \text{alt} - .548\ \text{grad} - .274\cos\theta - .365\sin\theta - 60.56$	.074	.111	.113
1	Planc	$= +4.062\text{alt} - 6.311\text{grad} + 6.76\cos\theta - 4.99\sin\theta - 1522.31$	.005	.006	.006
2		$= +1.096\text{alt} - 4.492\text{grad} + 8.16\cos\theta - 419.25$	.003	.006	.007
3		$= +1.413\text{alt} - 5.476\text{grad} + 3.03\cos\theta - 10.16\sin\theta - 577.16$	.014	.021	.022

Table 13 gives the results of multiple regressions. In predicting gradient, aspect adds little to the parabolic effect of altitude. The whole equation is quite different for Cache 1 compared with the other two matrices. Profile and plan convexity increase with altitude and decrease with gradient throughout, but aspect has little effect. The predictibility of profile convexity is low, and that of plan convexity is negligible.

The Cache area may be summarised as an upland fringe with broad summit convexities and convexo-concave slopes on the hills (Cache 2 and Cache 3). Gradient increases with altitude until the summit convexities are reached. Gradients are most commonly 1 or 2° in the lowland, and roughly 10° in the upland : none exceed 24°. Plan and profile convexity increase weakly with altitude, and aspect shows little relation to surface form. Second derivatives are difficult to interpret in lowland areas because of data problems.

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STATISTICAL CHARACTERIZATION OF ALTITUDE MATRICES BY COMPUTER. --ETC(U)
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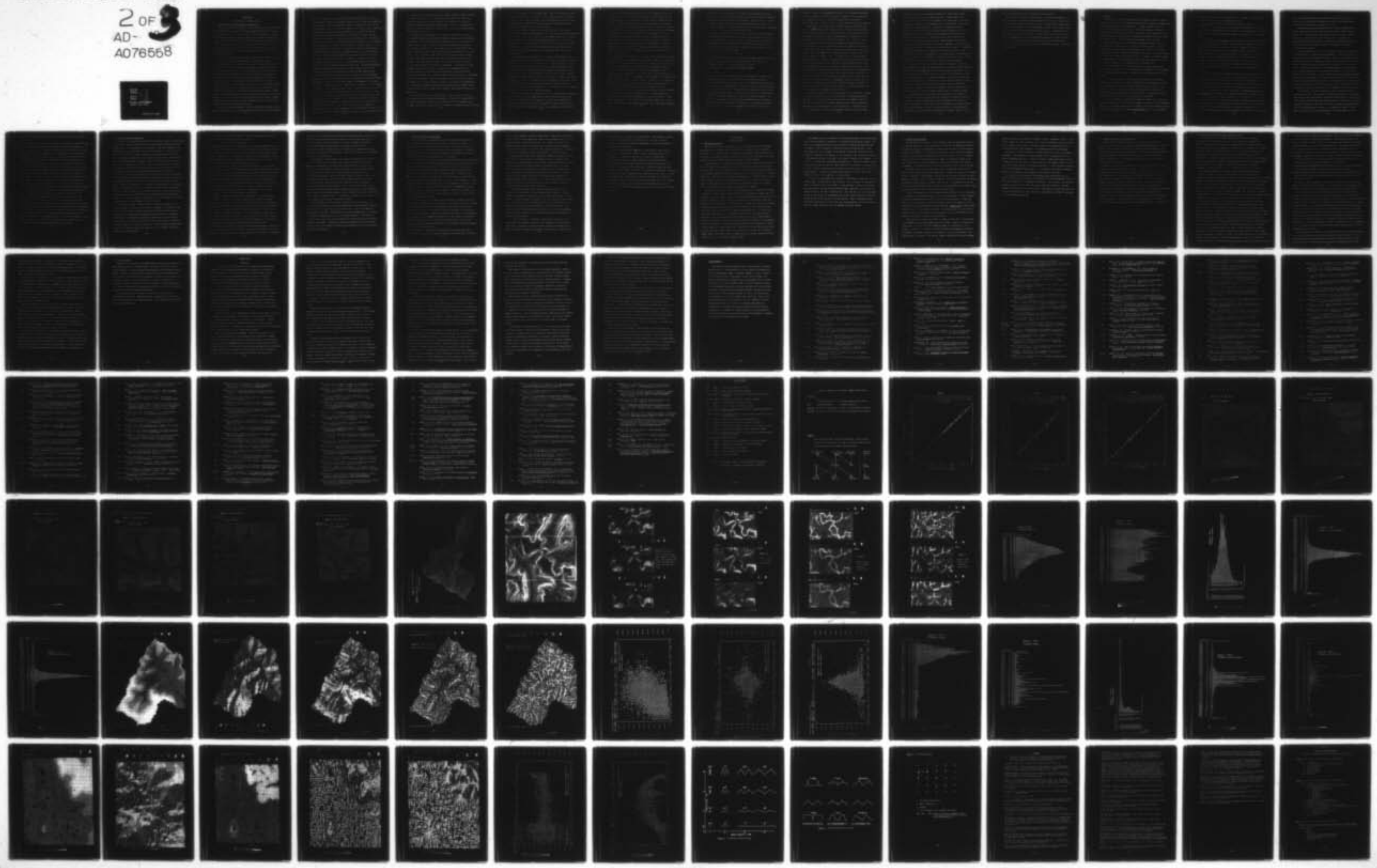

Chapter Six

OTHER APPROACHES TO TERRAIN ANALYSIS

(6a) Spectral analysis, filtering and autocorrelation

Rough surfaces contain variability at various spatial scales. By filtering the original data, it is possible to produce generalized versions of terrain; this is especially useful for 'noisy' data. It may be interesting to separate 'rough' and intermediate components of variability from 'smooth' by applying high, low and bandpass filters, or by repeated filtering (Tobler, 1969a): this is most easily and reliably achieved if data are in altitude matrix form. In this way 'microrelief' and mesorelief' could be separated; but no convincing demonstration that there is a clear-cut distinction between these categories has yet been provided.

Rather than presupposing that variation occurs at certain distinct scales, it is possible to apply spectral analysis to distinguish the strength of variation at each scale (for classes of scale, each covering a certain band of wavelengths). This is facilitated by the processing of equal-spaced points defining long terrain profiles. In spectral analysis, a profile is viewed as resulting from the superimposition of sine (and cosine) curves with various wavelengths, each of which has its own amplitude and phase. Such a model is natural for many time series generated by superimposed periodic processes (especially in electrical engineering) and for the study of ocean waves (Longuet-Higgins, 1962), but there is no process justification for its application to arbitrary terrain profiles. To interpret such profiles as the result of superimposed oscillations contradicts all that we know about processes of erosion and deposition. Spectral analysis of arbitrary terrain profiles is purely phenomenological, and gives no hope of a process interpretation, nor can the results be used directly to predict geomorphic processes.

Nevertheless, spectral analysis of a profile provides four parameters which may be of descriptive value. These are (Pike and Rozema, 1975); (i) the magnitude

and (ii) the spacing of any marked periodicities along the profile; (iii) the contrast between longwave and shortwave variability; and (iv) the total variability in the spectrum. Pike and Rozema (1975) provided a comparison of five profiles for wavelengths between 240 and 3,700m, together with a thorough review of previous one-dimensional spectral applications; Yagodina (1975) analysed 69 profiles. Because of various preprocessing operations (windowing, filtering, tapering) necessary for the analysis of finite, discrete data (Rayner, 1971), the analyst is involved in subjective rule-of-thumb decisions; the variability in the spectrum may be considerably less than the total variability of altitude (Report 2). The loss of information is such that it is inadvisable to perform manipulations in the spectral domain, for example to calculate derivatives; after transformation back to the spatial domain, considerable distortion will have been introduced. This applies to both digital and optical operations. Although various rules have been proposed, the final choice of the parameters depends on a subjective trade-off of sidelobe suppression against variance loss. Considerable experience is required before spectral analysis can be properly applied, and different researchers will make different decisions even for the same spatial series.

Since replicable periodicities are rare in studies of natural topography, and even in the study of engineered surfaces, characterization of spectra involves principally (iii) and (iv). Over a given range of wavelengths,(iv) (the total variability) is proportional to the average height of a spectrum, i.e. the variance. In relation to (iii), Sayles and Thomas (1978a) claim that the slope of a plot of log (variance) against log (wavelength) tends to a universal value of 2. Their evidence for this, reproduced in Thomas and Sayles (1978), spans seven and a half logarithmic decades of wavelength, from 4μm to 100m, and consists of data from various engineered surfaces, from runways and from six natural microreliefs (four terrestrial and two lunar). There is no doubt that this vast compendium of spectra gives an overall slope of approximately 2.0, but as Berry and Hannay (1978) point out, the slopes of individual spectra (covering up to four decades, and usually

two) vary from 1.1 to 3.0 and cover the whole range of possible 'fractal' surfaces (discussed in section (6b)). In reply, Sayles and Thomas (1978b) admit this variation but emphasise the central tendency; the mean slope is 1.98, and 10 of their 23 data sets have spectral slopes between 1.9 and 2.1. Mandelbrot(1975) shows that a Brownian sheet gives a spectral slope of 2.0, but he expects natural terrain to give 2.4 or 2.5.

The result is best interpreted by considering spectra which would result from superimposing similar waveforms, with amplitude varying in direct proportion to wavelength so that the mean gradient was identical for each waveform and only dimension (scale) varied. A plot of log (amplitude) against log (wavelength) would then have a slope of 1.0. Since variance (P.S.D.) is proportional to the square of amplitude, a plot of log (variance) against log (wavelength) would have a slope of 2.0. Hence the "common natural law" discovered by Sayles & Thomas (1978a) is simply that the shape or gradient of waveforms at different scales is constant. The scatter about their $\alpha = 2$ line covers two decades of variance, i.e. one decade of amplitude in relation to wavelength. The variation in arctan (gradient) thus implied is tenfold. Because of their standardisation (by k) procedure, gradient variation between data sets has been removed: the tenfold variation is within data sets. Sayles and Thomas (1978a) have therefore demonstrated that, over two logarithmic decades of wavelength, amplitude is likely to vary by two decades, plus or minus one decade. However, their evidence of sloping spectra for 23 data sets is important, since surface modellers have often assumed 'white noise (flat) spectra up to a sharp cut-off frequency (Whitehouse and Archard 1970; Nayak 1971).

The four terrestrial microreliefs included in that study are a lava flow, a micro-badland, and two military proving grounds. These are not representative of the earth's surface; they contain much more variation at short wavelengths than is common. Some of these spectra are taken from analyses by Rozema, and they can be seen

more clearly in figures 7 to 9 of Rozema (1969). There it is evident that their slopes are steeper than -2.0, especially if the convexity around 50m wavelength is ignored. This convexity may be due to the detrending procedure used to remove longwave variation: Rozema (1969) does not state the length of his profiles, though this now appears to be of central importance.

For the approximately straight section of Rozema's (1969) log-log plot, from 20 to 2m wavelength, spectral gradients for four lunar profiles are approximately -2.25, -2.50, -2.55 and -2.80. For both the Perth Amboy badlands and the Bonito lava flow, they are -2.25; for the Aberdeen proving ground, -2.7; for the Yuma proving ground, -3.0; for the Cinder Lake crater field, -3.0; and for the Suffield test crater, -3.25. Clearly the latter, steeper spectra imply more difference in form between spatial scales, whereas the badlands and lava-flow are more scale-independent.

The spectral slopes as calculated by least-squares linear regression (with the x axis reversed) by Pike and Rozema (1975) for their five contrasted mesorelief profiles are -2.06, -2.16, -2.35, -2.93 and -3.09. These profiles too are not a representative sample, but they do show that in terrestrial topography variance is likely to be more concentrated at longer wavelengths than is implied by the similar waveform model. Pike and Rozema (1975,p.512) actually claim, following Lettau, that "An overall spectral slope of -3 is thought to indicate a "uniformity" of topographic slope for all features in an area, regardless of their size." From our reasoning above, and from the results of Sayles and Thomas (1978a), this appears to be false: -3 should be replaced by -2. The error may be due to confusing labelling of the vertical axis of the power spectrum.

Related to the analysis of power spectra is that of autocorrelation functions, plots of correlation between lagged pairs of data points as functions of lag. This too is common in the study of engineered surfaces, where (among others) Thomas (1975,p.205) has asserted that "the geometry of surface profiles can be specified completely by two parameters, the r.m.s. roughness and the correlation

length" (see also Whitehouse and Archard 1970: r.m.s. roughness is simply the standard deviation of the height frequency distribution, providing a measure of vertical dimension). The concept of correlation length is based upon near-monotonic (commonly exponential) decline of autocorrelation with increasing lag. There are several competing definitions; correlation length (or distance) is the distance required for autocorrelation to decline either to $1/e$ (Whitehouse and Archard, 1970) or to 1/10 (Thomas, 1975) of its initial value of +1.0. In a geomorphologic application, Drewry (1975) has independently defined an "autocorrelation distance" as the lag at which autocorrelation declines to a given confidence level "usually taken as 95%". Interestingly, he pairs this parameter in his study of Antarctic subglacial morphology with the standard deviation of heights, giving a 2-parameter characterization comparable to that used by many engineers.

Unfortunately, the implication of the linear slope of most spectra is that differing high-pass filtering (detrending) procedures change the autocorrelation function: this has been demonstrated by Thomas and Sayles (e.g. 1978, fig. 4b). The correlation length therefore increases with profile length, as does the variance of heights. Autocorrelation functions are meaningful only on the same assumptions as power spectra. Sayles and Thomas (1977)have proposed use of the 'structure function' (cf. Matheron's semi-variogram; see Huijbregts, 1975), based on the sum of lagged differences instead of products; this is independent of the mean plane and may deal better with some types of nonstationarity. It has yet to be demonstrated, however, that this is a general solution.

A further claim contained in Sayles and Thomas (1978a) is that since spectra of profiles of surfaces have a universal slope of 2, a single parameter, k, "uniquely defines the statistical geometry of an isotropic surface for a given range of wavelengths". k, the 'topothesy', has units of length, i.e. it is a scale parameter; shape has been assumed invariant. This single-parameter approach

comes from a highly generalising view of surfaces, and represents backsliding from the view that two independent parameters (vertical and horizontal dimensions) are required to characterize unworn model surfaces. Shape is likely to vary and, as Thomas and Sayles (1978,p.167) admit, further parameters are required for worn (run-in) surfaces. For natural surfaces, even more parameters will be required; we will return to this topic in the concluding chapter.

Since the topothesy, k, is the only scale parameter in Sayles and Thomas's (1978a) approach, it must measure vertical as well as horizontal dimension, the two being assumed proportional to each other. In other words, it is a measure of roughness and, like the variance, will be greater for longer profiles on the same surface. So will the substitute T proposed by Berry and Hannay (1978) for the case of variable spectral slope, α:

$$T = k^{1/(3-\alpha)}$$

All of these proposals fall victim to the lack of an upward wavelength limit on variability, which is but one form of nonstationarity. How, then, can a representative measure of horizontal dimension be obtained?

Whitehouse (1974) has proposed an average wavelength

$$\lambda_a = 2\pi R_a/\Delta_a$$

where R_a is the mean height deviation and Δ_a is average slope. Likewise Tabor (1975) has shown that, for several models of surface contact, the onset of plastic deformation depends on average slope. In the Greenwood-Williamson-Tripp model, slope is given (approximately) by (the square root of) the mean deviation of asperity heights divided by the (constant) radius of curvature of asperity tips: in the Whitehouse-Archard model, it comes from (the square root of) the standard deviation of the height distribution divided by the correlation distance. In other words interdependence of the three profile parameters vertical dimension, horizontal dimension and mean slope, most apparent in a 'sawtooth' model profile, can always be used to find the third parameter, given any two; there are two

degrees of freedom in such simple profile models. It is therefore unnecessary to resort to spectral analysis (which gives two parameters, the average height and the spectral slope, for a linear sloping spectrum) to characterize a profile; the same information is implicit in mean gradient and vertical dimension.

Consideration of this engineering literature therefore supports my case (Evans 1972, & Rept. 2) against spectral analysis, and buttresses use of the moments and derivatives proposed here. (The use of moments of the height distribution is frequent in studies of the wear of engineered surfaces, especially by Endo and Kotani, 1973 and by Stout, King and Whitehouse, 1977. The use of slopes and curvatures is also found; see the bibliography by Thomas and King, 1977). Although slope and curvature distributions are, as both engineers and geomorphologists have found, highly sensitive to sampling interval (spatial resolution), they are almost insensitive to the longwave variations ('trends') likely to be found in geomorphic surfaces. Hence it is more essential to specify sampling interval than length of profile. Spectral analysis and autocorrelation methods provide parameters which fail to be intrinsic surface properties for the quite different reason, that there is no upward wavelength limit to variation. They are insensitive to sampling interval because variance at short wavelengths tends to be much lower: the observed slope of terrain spectra reduces the problem of aliasing. For spectral and autocorrelation parameters it is therefore desirable to specify the sampling interval, but essential to state the length of profile and the various preprocessing operations. The spectral model is unnatural for geomorphic data, and is sensitive to various types of nonstationarity which are likely in spatial data (Granger, 1969; and Report 3).

Preference for moments of derivatives is based on their more direct relevance to geomorphic processes, as well as the possibility of avoiding preprocessing operations which involve many subjective decisions. As I wrote in 1972, both 'spectral' and 'derivatives' techniques "probably produce similar information... They differ in the information which they make explicit,..."(Evans,1972,p.36).

The sensitivity of spectra to nonstationarity makes it difficult to apply

mathematical studies on the statistical properties of rough surfaces viewed as realisations of stationary stochastic processes (Longuet-Higgins, 1962 ; Nayak, 1971); these are also heavily dependent on the assumption of a Gaussian distribution of heights. It is remarkable that these unrealistic models, which cannot hope to capture the wealth of variability of geomorphic surfaces, nevertheless provide the basis for tolerable statistical predictions of, for example, the diffraction of echoes and electromagnetic waves (Beckmann and Spizzichino, 1963; Berry, 1973). These theories are said to work well for periodic and for relatively smooth surfaces, though Beckmann and Spizzichino (1963, p.410) do admit that "terrain is less easily described statistically than the surface of the sea; the variance of the results is very large...". Presumably these diffraction phenomena are insensitive to differences in surface shape which are of importance in geomorphology. For similar reasons, the mathematical models of Grenander (1975; Freiberger and Grenander, 1977) are difficult to relate to real geomorphic surfaces and to process studies by geomorphologists.

So far, attention has been focussed on profiles. Extension to surfaces greatly magnifies the technical and geomorphological problems of spectral analysis. Harbaugh and Merriam (1968) applied double Fourier series, while Rayner (1971, 1972) calculated two-dimensional spectra for several altitude matrices and gave full technical details. In his program, variance is calculated for square cells in the 2-D spectral domain. Because of the rapid increase in variance with wavelength, it is almost impossible from these results to assess differences in variance with orientation, within a given waveband. There are also problems of outward 'leakage' of variance along the principal axes of the spectral domain.

A spectral model cannot cope with several properties of real geomorphic surfaces, such as curving ridges (which blur directional attributes of spectra) and ridges of steadily changing width between converging valleys (which blur spatial frequencies). Spectra vary not only with geology but also with position in the erosional system, so that it is almost impossible to meet stationarity requirements even after trends in the mean have been removed. Spectra of most

natural terrains seem to vary continuously in their properties.

Where terrain is lineated, it may be better to calculate one-dimensional spectra separately along the grain and across the grain (e.g. for a drumlin field or for structurally-controlled relief). Even so, a profile must be long enough to include numerous examples of all the wavelengths important in that context, without crossing a regional boundary and mixing different spectra. Some regions are not large enough to provide such profiles. Spectral analysis is much less useful for general geomorphometry than for the specific geomorphometry of carefully chosen profiles (e.g. along a river bed). Finally, it should be noted that spectral analysis is undirected: it deals (in 2-D) with orientation, not aspect, and cannot handle asymmetry.

(6b) Fractals

B.B. Mandelbrot (1967;1977) has assembled and integrated a number of diverse mathematical concepts and observations into a model which he applies to geomorphology, hydrology, astronomy, and blood circulation. In terms of graphics in his 1977 volume, geomorphology has pride of place. This synthesis poses a challenge to geomorphologists to analyse data in such a way that the realm of application of the fractal concept may be defined, and to demonstrate deviations from what must be an oversimplified model. As yet, they have not responded to this challenge.

Mandelbrot's synthesis does not provide an alternative system of landform analysis; the basic fractal model is istropic and has two parameters, and so it can express only certain aspects of land surface form. However, it is the most realistic surface model to emerge from mathematics, and hence is of special interest to those who wish to generate 'synthetic terrain'. Further, it is highly relevant to the sensitivity of terrain properties to the spatial resolution at which they are measured: therefore it will, exceptionally, be considered here.

A very brief summary of the fractal concept will be given, starting with its application to coastlines. It is necessary to study Mandelbrot's illustrations fully to appreciate the relevance of fractals but the following basic notions are involved.

(i) A (rocky) coastline is so irregular at all scales that its length is defined only with respect to the unit increment η with which measurement is made. Data from Richardson (1961) are quoted to demonstrate a linear increase of log (length) as log (η) is decreased, mainly over the range from 1,000km to 30km (with one further data point each at 2,000km and 10km). A considerable amount of evidence reported by Volkov (1950) also supports this observation.

(ii) However irregular, a line has the topological dimension 1 . Models can be devised, however, of lines which are so irregular that they fill the 2-dimensional space in which they are embedded; such, for example, is the case for Brownian motion. It is useful to attribute such lines a <u>fractal dimension</u> , D, of 2, to supplement

the notion that their topological dimension is 1. In general, a fractal is a set for which D strictly exceeds the topological dimension.

(iii) The length of a fractal line, measured with unit increment η is $\lambda \eta^{1-D}$. 1-D turns out to be the gradient of Richardson's log-log plot, while λ is an approximate measure of extent of the line in dimension η and has the useful property that unlike any measure of line length, it is independent of η.

(iv) Fractals are self-similar, that is, invariant to certain transformations of scale: an enlargement of any part bears a statistical resemblance to the whole. Mandelbrot's assertion that fractals are realistic models for coastlines and for relief therefore implies that no scale-dependent phenomena are involved. In two places, Mandelbrot (1977) hints at limitations to this assertion. On page 44, a constant D at scales of interest in geography (1m to 100km) "does not exclude that in the range of sizes of interest to physics, the coastline may have a different D."; and on p.24, the loss of very large and very small scales is admitted; "Almost everywhere in Nature, both cutoffs are either present or suspected."

(v) Land surfaces are also irregular and thus have a fractal dimension in excess of two, but (much) less than three. One possible model for such surfaces is the Brownian plane-to-line function; this exhibits global interdependence and avoids self-intersections, and could be accounted for by a random faulting process. However, it has D=2.5 and is too irregular to represent most natural surfaces. Smoother (more 'persistent') surfaces are generated by fractional Brownian plane-to-line functions : that with D=2.25, though in some ways too irregular, is the most realistic exhibited to date (Mandelbrot 1977, top of pp.210 & 212). The same function provides D=1.25 for its coastline, which is the maximum value suggested by Richardson's data. Mandelbrot derived a further value of D=1.3 for coastlines from Korčak's law for the size-frequency distribution of islands; here D is a measure of fragmentation, rather than the irregularity of a continuous line. Finally,

there is some analogy between these values and a Hurst coefficient, H, of .8 for time series such as river discharges which exhibit long-term persistence, since these can be modelled by fractals with $D = 1/H$.

The geomorphologist must immediately point out several *landforms where* scale is important; sand dunes (Wilson,1971) and possibly drumlins (Rose and Letzer 1977). Self-similarity cannot apply to these: but it must be admitted that there are few rigorous studies of landform scale in geomorphology. Geomorphologists should feel prompted by Mandelbrot to demonstrate exceptions to self-similarity, since these would show the importance of scale-specific processes, past or present.

Studied at higher resolution than 30km, coastlines are not a promising field for self-similarity. Firstly, they are polygenetic. Secondly, we expect both geologic structures and geomorphic processes to operate at characteristic scales. For example, the width of landslides increases with the height of cliff; if the latter is constant, coastal irregularities will be produced at a well-defined scale. Sandy beaches are smooth at scales from kilometres to millimetres, save for the possible presence of beach cusps which are highly periodic and scale-specific, hence non-fractal. A fractal by definition has no determinate tangents, and therefore no orthogonals; but orthogonals to beaches have very interesting process implications (Davies, 1958, 1972). Since beaches in coves often alternate with rocky headlands, to restrict fractals only to rocky coasts at still water, as Mandelbrot (1977,p.28) wisely does, is a considerable retreat from generality.

In the absence of relevant data,however, these observations constitute whistling in the dark. The existence of digital 'world data banks' of coastlines makes extension and amplification of Richardson's results easy, and finer scales can be studied from large-scale map sheets which exist in digital form; no further digitising is required. This, then, is a further challenge to geomorphometrists, and this challenge extends to surfaces. It is urgently necessary to obtain empirical values which could be interpreted as the fractal dimension of land

surfaces, supplementing Mandelbrot's visual (and sometimes poetic) calibration of his models.

This discussion is intended as a constructive response from a geomorphologist to the exciting possibilities presented to geomorphology by a mathematician. It is of course the duty of a scientist to consider the realism of such models with a more sceptical eye than a statistician. Unlike Gaussian or Brownian models, fractals with dimension D between 2.0 and 2.5 succeed in modelling the spatial 'persistence' of real topography. They omit the organising tendencies due to fluvial and other processes which tend to give more continuity of slope, and to infill or open out closed depressions; hence in many areas depressions are much less frequent than closed summits. The question may be asked, whether geomorphologically reasonable and general surfaces can be modelled without taking the linear agency of rivers into account? Fractal models are purely phenomenological, with no relevance to earth surface processes. The model of scale-independent random faulting invoked by Mandelbrot, or even a model of random uplifts, is of negligible interest since it is infeasible both geophysically and geomorphologically.

Nevertheless, the fractal models remain the most realistic available; this is a sad comment on the even greater lack of realism in other models. They therefore form a useful starting point for further work, especially in the generation of artificial terrain. Much of the criticism noted here is based on empirical periodicities (in drumlins, sand dunes, and beach cusps); this may seem strange after criticisms of spectral analysis in the previous section, emphasising the very rarity of periodicities in natural landforms (especially in two dimensions). Obviously, the truth must be that there are elements of periodicity in this largely fractal world. The two approaches should be reconciled: meanwhile, fractal models form the 'null hypotheses' from which invocations of periodicity and scale-specific processes must demonstrate significant deviation.

(6c) Other computer-based approaches

A number of other approaches require brief consideration: these are computer-based in that surface attributes are produced by a computer program operating on digital altitude data. The techniques are; (i) power series trend surface analysis, (ii) eigenvalues of altitude matrixes, and (iii) texture analysis. Older work such as that of Hobson (1967) and Turner and Miles (1968) was discussed in Evans (1972). For slope mapping, Lecarpentier (1974) has applied a system based, like Hobson's, on isosceles but non-equilateral triangles. The work of Hormann (1968,1971) and of Mark (1975a) is discussed in section (2c) above. An unpublished draft by Terman (1972) reported an analysis of altitude, gradient and curvature distributions for a 200m-mesh 6 x 6km matrix covering the Half Moon Bay area of California.

Low-order power series polynomial trend surfaces (where altitude is regressed on northings and eastings) have been fitted to subjectively selected altitude points, for example by Mommonier (1969). The fit of such models to the data is usually given in dimensionless terms as R^2, but this should be supplemented by the standard error of the estimate of altitude. For extensive areas of erosional topography, the fit of linear, quadratic or cubic surfaces is very poor, unless there is some underlying tectonic or constructional form and the data points are selected very carefully. For example, King (1969) and Beaumont (1970) have attempted to establish the present form of warped planation surfaces in this way.

King (1969) interpreted clusters of positive residuals as monadnocks, and cluster of negative residuals as early valleys or downwarps. Unfortunately, there is no reason why these two different effects should balance each other exactly, as required by the least squares fitting techniques, and there is a fundamental contradiction in interpreting such groupings of residuals: if the residuals are autocorrelated in this way, the procedure of fitting trend surfaces is seriously distorted (Tarrant, 1970; Unwin and Lewin, 1971). Where altitudes are sampled more objectively (Thornes and Jones, 1969), degree of fit is quite weak and there is strong spatial autocorrelation.

Higher order terms should be added until residuals can be treated as random noise; if this is not feasible,the trend surface model;

value = function + random error

is inappropriate. One of the early proponents of trend surface analysis wrote more recently that "Though widely used, it cannot be anticipated that polynomial or Fourier series have genetic significance for the spatial variability of most Earth science variables". (Whitten, 1975). Malyavskiy and Zharnovskiy (1973) exaggerated slightly when they stated that "Even for comparatively small areas, the actual surface of the earth is not described by elementary analytical expressions".

None of the studies to date has demonstrated that there is a clear contrast between 'signal' and 'noise', or between 'signals' at discrete spatial scales. Rather, trend surfaces have simply provided generalised descriptions of the broadest regional features of the land surface; similar generalisations can be obtained by empirical smoothing (Tobler, 1969a). Like spectral analysis, trend surface analysis seems more useful when applied to specific, readily-identified landforms such as alluvial fans and pediments (Troeh, 1965; note the use of polar coordinates) and raised beaches (McCann and Chorley, 1967; Gray, 1974). Broad-scale ('global') trend surfaces are of limited value within general geomorphometry.

Another technique used widely outside geomorphometry is eigenvalue analysis. When applied to correlation or covariance matrices, this is known as principal component analysis. Gould (1967, p.62-64), proposed and exemplified its application to triangular altitude matrices, doubled over to form square matrices, with numbers standardised in the range (0,1), and the identity matrix added "so that we have a common standard for comparative purposes". For a smooth plain, all eigenvalues other than the first are zero: this is modified by addition of the identity matrix, but Gould (1967) proposed use of the ratio of the first eigenvalue to the second as a measure of surface smoothness, possibly after extracting major trends.

This proposal is seductive to those who prefer complex approaches to simple ones, but it can withstand little critical thought. What does this intangible ratio tell us that is not evident in simple form from any measure of the variability of altitude?

Vincent and Poole (1978) pointed out several problems: in particular, addition of an identity matrix modifies the results (in general, it adds a high ridge along the principal diagonal) without preventing the occurrence of negative eigenvalues. It is not surprising that these authors "were unable to detect any clear pattern amidst either the eigenvalues or the eigenvectors of appropriate matrices..." constructed from theoretical terrain models. There is no justification for eigenvalue analysis of altitude matrices.

Some analogies to the problems of land surface form analysis may be found in picture processing operations, where altitudes may be treated as 'grey levels'. Weszka, Dyer and Rosenfeld (1976, and earlier reports) applied a variety of such operations to air photographs. They found that the co-occurrence matrix features of Haralick were no more efficient (in discriminating terrain types on the Carboniferous of Kentucky) than simpler features based on histograms of grey level differences at given lags; and that both were considerably more efficient than Fourier power spectra statistics, even when 'aperture effects' distorting the spectrum were removed (Dyer and Rosenfeld, 1976). The co-occurrence matrix cross-classifies the frequency of grey levels associated at a given lag. Mean differences between (Queen's case) nearest neighbours usually perform as well as any other features: they correspond to apparent slopes. In some cases, improvement could be achieved by local averaging of picture grey levels before analysis.

This endorsement of relatively simple operators is most encouraging, as is the statement "The textures of the terrain samples may be more appropriately modelled statistically in the space domain rather than as sums of sinusoids" (Weszka, Dyer and Rosenfeld, 1976 p.285). Although work in picture processing gives the impression of mathematical sophistication, it is in fact highly empirical. Those descriptors found most useful turn out to be crude estimators of the process-related descriptors proposed in this Report. Nevertheless, further developments in picture processing will be followed with interest since some may have implications for the more specific application considered here.

(6d) Other non-computer-based approaches

All the approaches considered above are computer-based from the outset in that computer programs operate on altitude data. A further set consists of approaches which may (Scott and Austin, 1971: Pike, 1972) or may not lead on to computer analysis of descriptors, which have been derived manually. Obviously, it is not possible here to review such work thoroughly, but generalities concerning some of the more important contributions will be considered.

As discussed in Evans (1972), the main problems with manual general geomorphometry are the multiplicity of different measurement operations, and the variability of the areas over which they are measured. From maps, Wood and Snell (1960) measured average slope, grain (spacing of major ridges and valleys), average altitude, slope direction changes, relief, and altitude-relief ratio. Grabau (1958) and Van Lopik and Kolb (1959) used modal slope and relief, number of steep slopes per mile, and characteristic plan-profile with 6,7,6 and 25 classes respectively. Plan-profile is based on a cross-classification of crest-peakedness (2 classes), % area occupied by highs (3 classes; cf. altitude-relief ratio and hypsometric integral), linearity (2 classes) and parallelism (2 classes) of highs, plus a separate class for 'no pronounced highs or lows'. Hence the system is based on seven variables, which are coarsely pre-classified.

Though widely quoted, the 'characteristic plan-profile' system is less often applied and would appear difficult to automate. The main problems are the coarseness of the classes which conceal local variability in many regions; the fact that different class limits will be relevant to different purposes; and the arbitrary nature of the definition of 'highs' and whether they are 'sharp'. Parry and Beswick (1973) adapted both this system and Wood and Snell's (1960) system to quantitative measurement from air photos, but devoted an inordinate amount of time for the definition of 'highs'; separate determination of each variable from air photographs is a very time-consuming procedure.

Though there is considerable overlap between the two systems, van Lopik & Kolb (1959) did not use grain or average altitude, while Wood and Snell (1960) did

not use crest-peakedness, linearity and parallelism. There may be some relation between 'slope direction changes' and 'number of steep slopes per mile', in which case nine distinct variables are involved. All of these are covered by the present automated system, if linearity, grain and slope direction changes are considered as functions of the plan curvature frequency distribution. Parallelism is given by the strength of the periodic regression of altitude on aspect.

Another approach comes from the commendable attempts by Speight (1958, 1974, 1976, 1977) to lend precision to land form description in connection with Australian land system studies. In 1974, Speight distinguished landform elements, of some 20m extent, from landform patterns of some 300m extent. The latter involve more subjective properties such as grain, relief and drainage network type, while elements are described by altitude, slope and context. In 1976a Speight described separate procedures for delimitation and description of landform patterns at 500m - 5km scale: 3 to 5 coarse classes were used for each variable, and excessive reliance was placed on basal and summit 'surfaces of accordance', which are very difficult to define.

In Speight (1976), many types of landform elements were grouped into 34 classes, and a large number of landform patterns were recognised. Landform elements are essentially 'slope facets' and in basing his measurements on these Speight is taking an essentially atomistic view of the land surface (see chapter 2c). The main problem is the considerable variation in spatial extent of both elements and patterns, within which geometric variability is ignored. While Speight considered regions and facets inescapable, I feel that scale-dependence makes calculation of gradient over variable areas dubious, and calculation of convexity almost worthless.

In terms of definition of variables, Speight's main contribution is his insistence on the importance of context, e.g. distance from crest and height above stream. Calculation of these contextual variables is important for the study of geomorphic processes, but requires a different type of program to the

present one, which is confined to local calculations. Slope length is, however, calculated in a program by Moore & Thornes (1976). This type of analysis straddles the transition from general geomorphology to the specific morphometry of slope profiles (Blong, 1975).

Finally, it may sometimes be desirable to make explicit certain surface features which are not measured directly either in the present system, or in these others. The degree of dissection of a surface into summits may be measured given a definition of summit magnitude as "the smallest vertical ascent necessary to climb a summit, starting from adjacent higher ground", i.e. "the height of a summit above the lowest pass on the highest ridge connecting it to higher ground" (Evans, 1974 p.394). The intensity of summits may be expressed by relating the integral of magnitude times frequency either to area, or to length of connecting ridge. The greater the intensity, the greater the dispersion of plan convexity, in the unlikely event of other things being equal. This attribute is difficult to automate because it relates to broad spatial position, but automation should eventually be possible for this and for all the other attributes discussed.

Chapter Seven

FURTHER PROPOSALS

(7a) Improvements in output

The form in which output is produced is, of all the attributes of the integrated system described in chapters 2 and 3, that which is most specific to the computing environment. It is always possible to hope for more expensive, higher-resolution, more flexible or more purpose-oriented output devices; yet there is a challenge in tailoring a system to produce acceptable results on available or common hardware.

Improved versions of all graphs and maps presented here could be produced on high-quality flatbed plotters (especially those with light-head projectors),on electrostatic printer/plotters or on laser-beam plotters. One of the latter has now been purchased by the Geography Department, and when installed it will facilitate production of improved shaded maps. Experience of this plotter has been gained in a different research project (Clarke, Dewdney, Evans, Rhind, Visvalingam and Denham, 1979), using it on a bureau basis. Better shaded maps could be produced now on the drum plotter, but this would be very slow because of the thousands of grid squares to be shaded.

It would not be unduly difficult to produce separations, for superimposed printing in different colours. These could be produced on a line printer or a graph plotter, or preferably on a laser plotter. However, the extra time and expense involved makes colour maps more suitable for long print runs for publication of final results, rather than exploration of new data sets. Two-colour maps would be especially useful in contrasting convexity and concavity; the class limits on such maps would differ from those used here, except that zero convexity would remain the balancing point. Two colours would also permit superimposition of, for example, drainage lines to provide a base map. Alternatively, two monochrome maps in two different colours could be superimposed to provide a bivariate map. This is a field which presents challenges to the map designer; bivariate and especially multivariate statistical maps tend to be excessively complex, looking pleasant but communicating little. Simplification of such maps may be achieved by selecting certain limited classes of each variable, e.g. slopes over 10° might be mapped together with extreme positive or negative profile and plan convexities, giving a map with $2 \times 3 \times 3 = 18$ possible combined classes.

Improvements can also be achieved with the existing equipment by writing additional special-purpose programs to replace use of the general-purpose package programs. The most obvious changes are those which are desirable for handling the large data sets (thousands or tens of thousands of values) provided by altitude matrices. First, the scatter plots could be shaded for the density of points (per printer position), instead of using digits. The shading, if still produced on the line printer, could be on the same basis as that for the maps : six classes of darkness could represent 1, 2-3, 4-6, 7-10, 11-15 and over 15 points per printer position. This would certainly create an improved visual effect, and the classification error is unlikely to be greater than that that incurred by grouping all positions with 9 and more values, as in the S.P.S.S. package program. If a graph plotter were used, there could be more classes, and a regression line (or R.M.A., see below) could be plotted.

Second, a minor improvement could be made to the histograms by applying a similar shading scheme. This affects only the last symbol, which at present can represent any number of values from one to the 'number of values per symbol' given by the automatic scaling procedure. Although this refinement may seem unimportant, it would give much better portrayal of the tails of distributions. More important (if a new program were written) is the inclusion of cumulated percent values for each class limit, from which percentiles and related statistics could be computed. Ideally, histogram programs should give class midpoints, class counts, and cumulated percent of total count; given pressure on space, percent per class is redundant and can be omitted.

(7b) Statistical improvements

The least satisfactory aspects of this integrated system are the moment statistics for convexity, especially in plan, and values of kurtosis for all skewed distributions. It might also be argued that regression is inappropriate for some of the relationships: but the only one where it is not possible to distinguish a controlling and a dependent variable is the relationship between profile and plan convexity. For this, a reduced major axis (R.M.A) would be more appropriate. Aspect is necessarily treated as the controlling variable in periodic regressions, and for other variables, the hierarchy of dependency (following the order of derivation) proposed in section (3e) may be used.

Calculated values of profile and plan convexity are meaningful insofar as the maps of these variables are clear and interpretable. However, the long tails of extreme values at both ends of these frequency distributions have excessive influence on moment statistics, especially of plan convexity. These tails are so pronounced that they weaken the defence of moment statistics offered in section (3e). Two possible solutions are the use of percentile-based statistics instead, or the transformation of convexity values in such a way that the 'tails' are brought in toward the centre.

It would be inelegant to use only percentile-based statistics for both convexities, while using only moment-based statistics for altitude and gradient. Yet mean, standard deviation and skewness serve very well for the two latter properties. A compromise might be to calculate median, inter-quartile range, several other ranges (e.g. 95 to 5 percentiles) and related skewness and kurtosis measures, together with the moment-based statistics, for all four properties concerned - altitude, gradient, profile convexity and plan convexity. This does however place greater onus on the interpreter as to the emphasis to be placed on each part of either version.

Transformation of convexity would have to symmetrical about zero : obviously none of the skewness-reducing transformations would be relevant. Where x is convexity as defined here, $(\frac{2}{\pi})$ arctan (kx) would change the range of convexities to -1 to +1, instead of -infinity to + infinity, thus making convexity more manageable in terms of both graphics and moment statistics. The constant, k, controls the severity of the transformation; the smaller k, the less the change in shape since the relation between

tangent and angle is near-linear at low angles. Either a 'reasonable' value of k could be held constant over a set of matrices, or k could be calibrated to give zero kurtosis in each frequency distribution. In the latter case, k would replace kurtosis as a descriptive statistic/model parameter. The consequences should be investigated.

Kurtosis would also be affected by a further alteration, to remove its dependence on skewness. This dependence is a major drawback, since it prevents kurtosis from being an independent descriptor of distribution shape. Skewed distributions, positive or negative, have one tail which is long in relation to the standard deviation of the distribution; hence they tend to have positive kurtosis. It would be useful to measure kurtosis after skewness had been removed : it could then be interpreted as symmetrical deviation in the length of tails compared with the breadth of the peak.

This might be done by applying a simple symmetrising transformation such as $\frac{x^{\lambda} - 1}{\lambda}$ (Box and Cox, 1964), where λ is calibrated to provide zero skewness. Kurtosis would then be measured for the unskewed distribution. λ would be highly correlated with skewness, and thus would not itself be used as a descriptive statistic. Existing programs (Dunlop and Duffy, 1974) perform this transformation for small data sets; they would require modification to deal with sets of the present size, probably by grouping all but the extreme observations.

(7c) Comparative studies at finer grid meshes

The grid meshes of matrices analysed here are essentially 25m or 100m: the 7.62m Gold Creek matrix is too small to permit thorough investigation. Coarser meshes have been studied by 'thinning out' these data sets, but studies at finer meshes require data at higher spatial resolution. These are now becoming available; the Gestalt company of Vancouver, B.C. have produced 4.56m - mesh data from an analytical stereo-plotter. Other high-resolution data sets have been produced especially for various projects.

Analysis of such data is not required in order to validate the present system; the existing results are sufficient. Extension to these finer meshes would, however, strengthen generalisations about scale variations in gradient and convexity. Mean and standard deviation of gradient, and the standard deviations of profile and plan convexity, increase over the range of resolution 2km to 25m. The change is apparently steady and continuous, though curvilinear. It would be interesting to see whether these changes continue down to 5m, where we are approaching the resolution of field measurements. Continuity is expected, since Gerrard and Robinson (1971) observed a comparable scale effect in varying the measured length for gradient in the field. Such an extension could also be used to test the robustness of the fractal model. From the point of view of geomorphology, this extension to bridge the gap between field-based and map -based measurements is of high priority.

(7d) Spectrum of variation of terrain properties in U.S.A.

If it is accepted that the properties defined above cover the essential attributes of land surface form, then it is clearly desirable to establish what values of these properties are likely to be encountered. This means establishing both mean or median values and measures of variability, and it requires either complete coverage or sampling on a probability basis. Lack of data prevents sampling of the whole world in this way, but it has recently become possible to obtain data for anywhere within a large and interesting subset of the world, namely for the conterminous United States of America (48 states). These data were produced by the Defence Mapping Agency Topographic Center (DMATC), and are being distributed by the National Cartographic Information Center of the U.S. Geological Survey (N.G.I.C., 1978).

The data were generated by digitising contours from stable-base prints of 1/250,000 scale maps, which have a 100-foot contour interval. Altitudes were interpolated in relation to distance to contours in four principal(axial) directions and to digitised spot height, stream and ridge line data; this was done for points on a square grid with .01 inch mesh on the map, that is 63.5 m on the ground. The results were stored in units of one degree longitude times one degree latitude, of which there are approximately 915; each occupies one eighth of a magnet tape. Each unit is a matrix of approximately 1750 x (1150 to 1600) points, depending on latitude and longitude (These estimates are not given in the official documentation, but are calculated here).

To process the matrices in their original form is not feasible with the present system and computer resources; map output would be cumbersome whatever device were used. Moreover, it would probably not be desirable to analyse the complete dataset, since the results for convexity would reflect the interpolation procedure used by DMATC. The grid mesh is dense relative to the contour spacing; this form of storage has a high degree of redundancy relative to its data content, and to describe the matrix as of 63.5 m mesh is to give a misleadingly high impression of its precision. Thinned versions may thus be more interesting to analyse. Taking every eighth point for a one-degree area would yield a manageable matrix with a mesh of 508m. Alternatively, a matrix of the same size could be generated by taking every fourth point for an area one-half times one-half degree, yielding a 254 m mesh. Both possibilities should be investigated, together with the behaviour of these data when pushed to their 63.5 m - mesh limit. For thinned matrices, four replicates could be analysed to assess the stability of results.

Even so, complete coverage of the conterminous U.S.A. would be extremely expensive: the main cost would be in computer processing time. Sampling about 50 areas would suffice to establish the range of results: this represents approximately 5% coverage if one-degree areas are used, or 1¼% for half-degree areas. The sampling pattern should not be completely random, since for a sample of this size coverage would then be uneven and major topographic regions might pass unsampled. The main alternatives are:

(i) systematic sampling, on a grid which is 'square' in terms of latitude and longitude: excluding a hexagonal grid as impractical given the square-gridded nature of the data, this gives the most even coverage which can be achieved;

(ii) locationally stratified random sampling, whereby one area is drawn at random within each four-degree quadrangle, and those falling in the sea are ignored;

(iii) stratified random sampling in relation to a sampling frame such as Hammond's (1964) landform regions; areas are drawn at random until a quota related to the area of each region is filled.

Of these, (ii) gains randomness at the expense of increasing the possibility that important landform regions go unsampled. (i) is preferred, the grid incidence being selected at random; but further investigation should precede the final decision.

There is bound to be a problem with edge effects, at the coast and at land frontiers, which affect 189 of the 915 one-degree quadrangles. If such areas were excluded, the sample would be biased against the peripheral areas. If they were included as drawn, the highly variable areas covered would make statistics difficult to compare with other matrices. If, however, peripheral samples were displaced just sufficiently to equal other matrices in land area, the displacement would produce bias in favour of peripheral areas. It is suggested that a decision between the latter procedure and total exclusion be made separately for each area by drawing random numbers, with the probability of inclusion proportional to the land area covered in the sample area as initially drawn. Hence the sample may be treated as representative of the conterminous U.S.A. This removal of bias is important because coastal areas are known to differ in topography from inland areas.

The result would be to establish how frequency distributions of altitude, gradient, aspect and both convexities, and relationships between them, vary over the U.S.A. No existing results permit statements about the expected range of variation in these properties, yet this information provides an important background for many types

of planning and of legislation. At present much environmental planning and law is enacted 'in the dark'; because environmental information is inadequate, the results and even the spatial incidence of such decisions are unpredictable. It is especially important for federal legislation and national military planning to anticipate the range of variation of environmental conditions.

Sampling the U.S.A. would permit full investigation of between-area relationships for summary statistics (chapter 3f) : since this requires a reasonable number of areas, with data generated in the same way, only the DMATC digital terrain tapes provide such an opportunity. This would permit study (by principal components analysis and multidimensional scaling) of the dimensionality of terrain properties, insofar as these are covered by the present system of analysis. The main limitation on the results is the coarseness of the 1/250,000 contours, but these data are the best available for such an extensive area.

Finally, the DMATC data provide the possibility of analysing terrain data for the whole U.S.A. in order to provide a regionalisation of landforms. This would be comparable to that of Hammond (1964), but based on a broader range of terrain properties and much more intensive measurement. Hammond took 4 classes of gradient (proportion of area gentler than 4.6°), 6 classes of range in altitude (per 10 x 10 km square), and 4 classes of profile character (proportion of the land gentler than 4.6° which is below the mid-range of altitude). He found that half of the 96 possible combinations were found over significant areas of the U.S.A.

The cost of applying the whole of the present terrain analysis system to the whole conterminous U.S.A., however, would be great, and consideration of such a project is best postponed until interrelationships and the spectrum of variability are evaluated by sampling, and the quality of the DMATC data is assessed. It might be that various shortcuts could be achieved by selecting a limited number of critical properties, while still making many more measurements than could Hammond in his manual study.

(7e) Offer to process

Meantime, it will be useful to accumulate further evidence on the variation of these properties, even on a piecemeal basis. This should be done for altitude matrices generated in as many different ways as feasible, and with as many different meshes. Therefore, I propose to invite scientists to send me suitable matrices. Single matrices or small sets would be processed on condition that the results could be tabulated along with others processed here, in addition to their use by the originators. Because of this need to build up experience in their interpretation, it is preferred that as much of this work as possible should be done at Durham, which also avoids the problems of software transference.

Suitable data sets should consist of at least 50 x 50 altitudes on an (approximately) square grid. Small matrices may be acceptable if included within a set of matrices generated in comparable fashion. Those interested should contact the Principal Investigator.

Chapter Eight

CONCLUSIONS

In conclusion, let us first return to the fundamental and controversial question : how many descriptors are required to encapsulate the variability of natural terrains? I would like to provide a definitive answer to this question from a broad and representative suite of analyses of different regions ; given the inadequacy of available data (section 3f), however, a discursive and deductive approach is all that can be offered at present.

An initial hypothesis is that more irregular surfaces require more descriptors. There are random process models which generate quite irregular surfaces from two or three parameters, yet the limitations discussed in chapter six imply that these models can only be very gross approximations to natural terrain and, indeed, they may omit important aspects such as non-stationarity, asymmetry, and the possibility of integrated drainage systems. We might propose a corollary of 'Murphy's Law'; over a broad range of earth surface topographies, what can vary, will...

Obviously we must start by mentioning the concept that one number will suffice to summarise surface roughness. This may be so for a very specific purpose, but a different number is likely to be required for each purpose. To assume perfect correlation between all measurable properties, over even a limited domain of surfaces, is so optimistic as to qualify for the description 'philosopher's stone syndrome' (Evans, 1972), or 'magic number syndrome'.

Engineers studying the microtopography of artificial surfaces have often used a single parameter (rms or cla) measuring relief (variability in height), but this is useful only <u>within</u> a family of surfaces produced by a single process. Within such a family, many other elements of surface form remain constant (section 6a, and Posey, 1946). More recently, relief has been supplemented by autocorrelation distance, and Whitehouse and Phillips (1978) now propose a pair of descriptors for the autocorrelation function; together with relief, these form a trio.

The use of two or three descriptors may be of value for theorising, but a more realistic approach was proposed many years ago by Posey (1946). I have recently found that he proposed a synthesis comparable to mine, except in being related to profiles instead of surfaces. He proposed that the frequency distributions of profile height and its first two derivatives, (apparent) slope and (apparent) curvature "characterize the roughness of a profile quite completely", whether it be regular or not, so long as there are no overhangs. His useful illustrations show profiles whose distributions vary in skewness and kurtosis, implying that quite a number of descriptors would be required even for profiles. Myers (1962) found autocorrelation inadequate and power spectra unnecessarily complex; he followed up Posey's ideas by proposing use of standard deviations of (1) height (2) algebraic slope and (3) apparent curvature, plus (4) a measure of profile asymmetry. (He found that (2) correlated best with friction between surfaces).

Taking a deductive approach, we may start with a two-parameter sawtooth model profile, widely used in the engineering literature (Fig.43). The horizontal dimension (wavelength) and the vertical dimension (amplitude) can in theory be varied independently; high-amplitude shortwave forms seem unlikely in nature, but are found in pinnacle karst such as that of Gunong Mulu, Borneo. Under this rigid model, and under a comparable sine wave model, the average absolute slope is exactly predictable from the two parameters, wavelength and amplitude; when these are plotted against each other, lines of equal average slope radiate from the origin.

Although in engineering applications the average height of a profile is usually arbitrary and is set to zero, in geomorphology absolute altitude above sea level is important, giving a third independent descriptor. Further dimensions of variability are easily envisaged (Fig.44). For constant amplitude and wavelength, a greater proportion of the profile can be concentrated either at high or at low heights : this fourth property (massiveness) is measured by the skewness of height. Fifthly, the homogeneity of heights can be varied : of much of the profile is near the maximum and minimum heights, kurtosis is negative; while if most is near-

average, with occasional spikes and pits, kurtosis is positive (Fig.44b).

Next, the 'waveforms' considered above may, while retaining all their other properties, be more or less asymmetric : the right-facing slopes of a profile may differ from the left-facing (Fig.44c). Hence we have a six-parameter model for profiles of simple, repeating waveforms : five parameters for form, and one for vertical position. Yet instead of every hill and every valley being an identical repetition of its neighbours, they may vary in amplitude, wavelength, massiveness, homogeneity and asymmetry. This might double the number of parameters required : whether the resulting twelve profile parameters are increased by complex interactions or decreased by intercorrelation (redundancy) has yet to be demonstrated.

Proceeding from profiles to surfaces, the same number of parameters will suffice only for isotropic surfaces. (Nayak (1971, 1973) has shown, however, that the parameters of an isotropic surface differ from those calculated for random profiles across it). If, for a non-isotropic surface, profiles in two perpendicular directions could vary independently, the number of parameters required would double : such independence,however, seems inherently unlikely. At the very least, extra properties such as degree and orientation of lineation,and direction of asymmetry, are possible. Even for Gaussian surfaces, Nayak (1971) stated that disregarding position, three invariant parameters are required for isotropic surfaces, or seven for non-isotropic.

For real, non-Gaussian, non-isotropic, irregular surfaces it is not unreasonable, then, to start with four descriptors for each derivative, and ten for their interrelations, making thirty in all. (Of these, we have omitted measures of skewness and kurtosis of aspect in the present program suite). There is surely some redundancy among these; for example, skewness of gradient should relate to the modulus of skewness of altitude. There are unlikely to be between-area correlations of $\pm .99$: rather, one descriptor may be predictable from a group of others. Until this redundancy is clearly demonstrated, the full range of descriptors should be retained. It cannot be proved that all are required, but this should be assumed until there is strong evidence that some are not required. Within certain

types of terrain, some may be irrelevant; as we have found, many within-area correlations are near-zero.

Taken together, these descriptors cover most surface properties. There is no explicit measure of wavelength, but this is implicit (other descriptors being equal) in mean gradient and standard deviation of altitude. Likewise, perhaps summit intensity can be predicted from a group of these descriptors. I maintain, however, that redundancies and intercorrelations are unlikely to be so strong that factor analysis will establish 'fundamental dimensions of variability'. Factor analysis is more likely to obfuscate and confuse by making arbitrary combinations based on moderate correlations.

Engineers given the task of quantifying land surface form are naturally inclined to use spectral analysis, which has proved useful elsewhere in analyzing irregular profiles. Quite apart from practical difficulties such as the need for very long profiles in each direction, this technique is proving unsuitable for the complexities of natural terrain, especially with the general dominance of longwave variability. Even for engineered surfaces, dissatisfaction with spectral analysis has been expressed of late (Thomas and Sayles, 1978); natural surfaces are more complex because they are subject to a broader range of interacting processes.

The advantages of the present approach lie in its simplicity, directness and conceptual economy. It starts from the properties of points on the surface, and shows how (excluding position) the properties of geomorphologic and human interest may be measured by derivatives of the surface. Without defining any elaborate indices or using indirect analyses such as variance spectra, areas may be characterized by moments of the frequency distributions of 'point' values, and by correlations between 'point' properties. In this way, the summarisation of areas has been coordinated with the mapping and interrelationship of the 'point' properties.

Hence it is maintained that the aims of this project have essentially been fulfilled, and an attempt has been made to place the proposed techniques in a broader conceptual context. An integrated system which provides both graphical displays and numerical summaries has been developed. From a single data input - the altitude matrix, with rapidly improving availability - maps, histograms and interrelationships are portrayed for the 'point' values altitude, gradient, aspect, profile convexity and plan convexity. Numerical summaries include moments of frequency distributions, correlation coefficients and selected regressions, and these characterize the areas included.

The main grid meshes investigated here are 25m and 100m. The latter suffices for areas of glaciated mountains because these are dominated by broad-scale (longwave) landforms. 25m is desirable wherever possible, and finer meshes are of interest in permitting comparison with measurements made in the field, commonly over 1.52m to 5m slope lengths. The Gold Creek matrix, less than 1km across, is too small to provide descriptors representative of a broader area. Each of the three Cache matrices is 2.5km square, providing areas which seem inadequate to characterize land surface form in that region; results vary greatly between these adjacent matrices. The Thvera matrix covers 10 x 10km, and its four quadrants are just about large enough to provide similar summary statistics (see section 3c). It seems, then, that areas of at least 5 x 5km will be necessary to encompass most of the land surface properties of a region.

Results are inevitably sensitive to the grid mesh of the initial data, but the moment descriptors are less sensitive (at least in part) to the extent of area covered than are autocorrelation and spectral techniques. Coarse, inflexible class intervals are avoided: the approach is fully quantitative, yet quite straightforward, providing results which are directly meaningful to geomorphology and to human activity. As experience in the use of this comprehensive system is built up, the need for further improvements or simplifications may become apparent; but it has now been developed to the point where it should be broadly applied.

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n.b. In the computer titles, 'calculated height' is equivalent to 'altitude', 'curvature' is equivalent to 'convexity'.

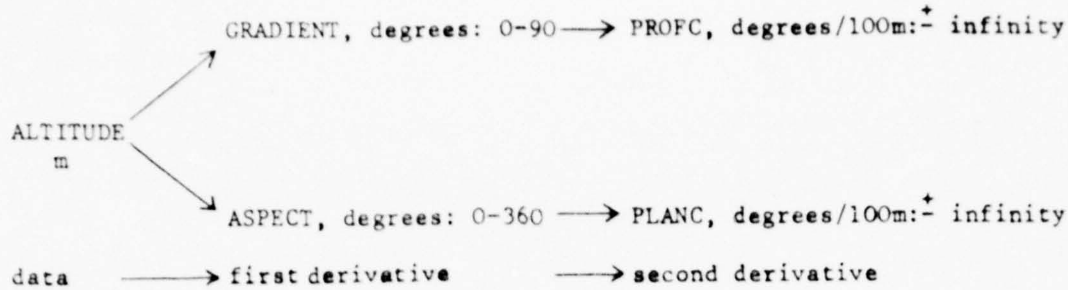


Figure 1 The five basic properties of a point on a single-valued continuous geomorphic surface: altitude and its two most important spatial derivatives.

Figure 2

Data sources and types of surface form analysis. Arrows indicate the most common derivations, although all combinations are possible. Double arrows denote increasingly important routes.

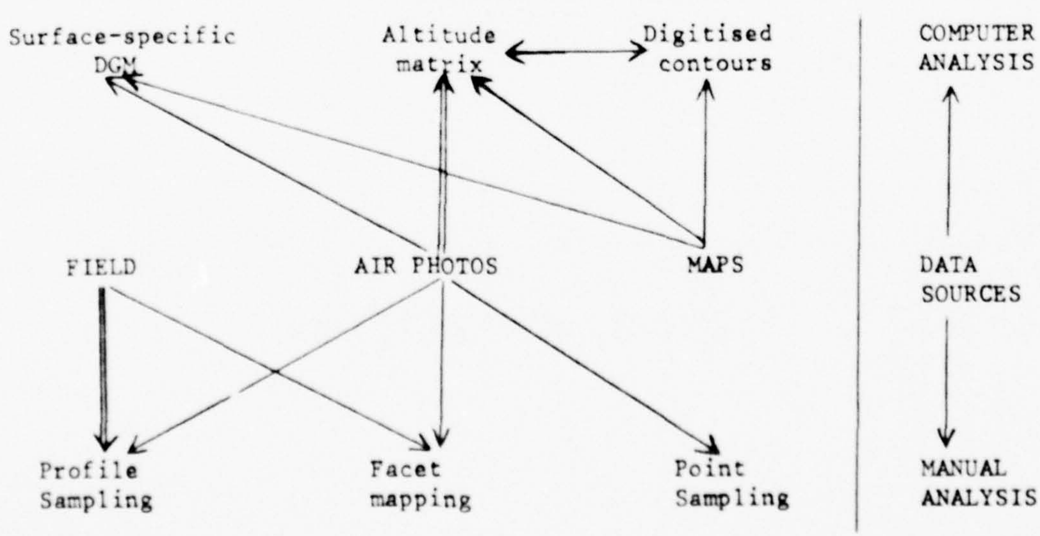
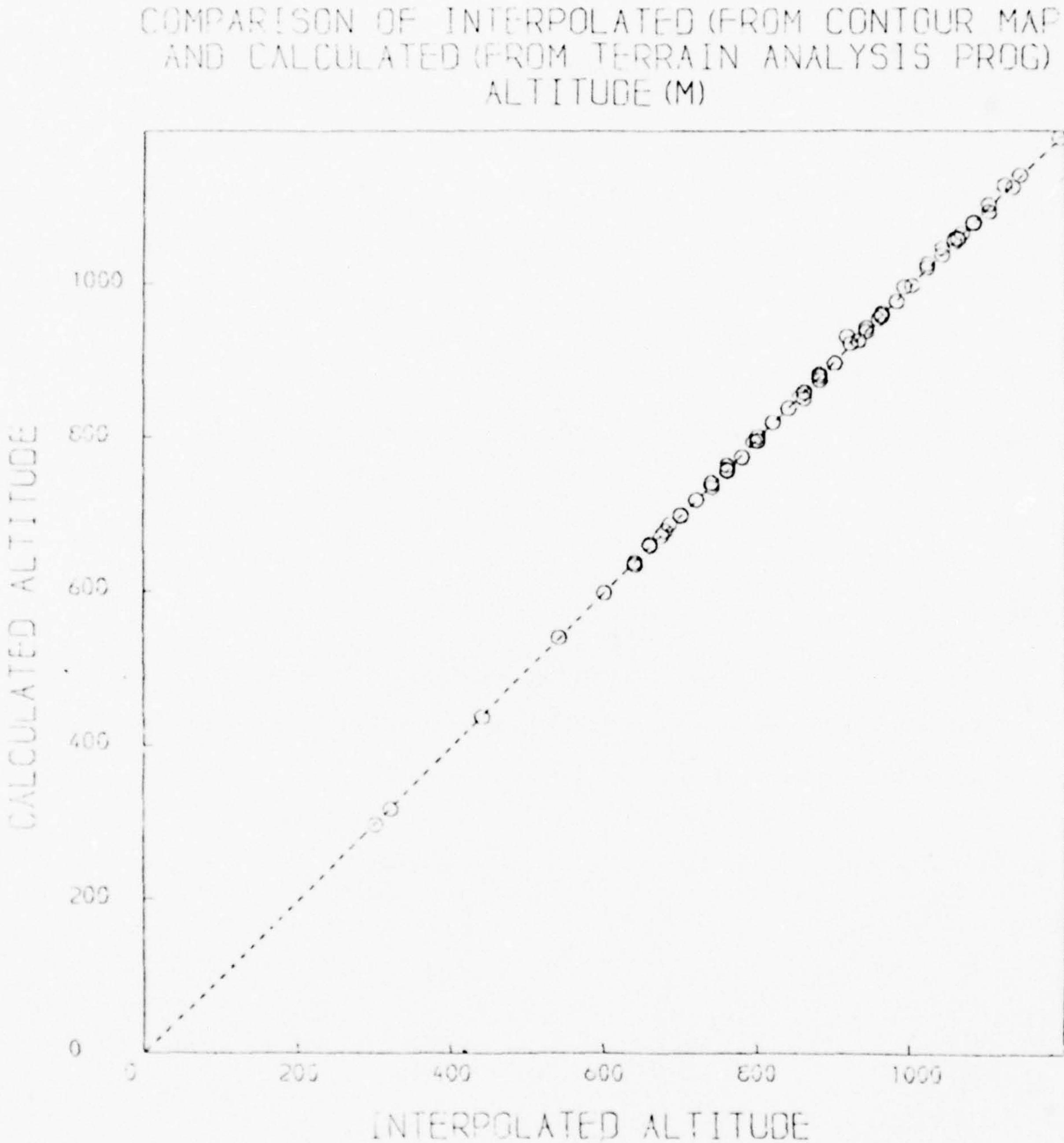


Figure 3



COMPARISON OF INTERPOLATED (FROM CONTOUR MAP
AND CALCULATED (FROM TERRAIN ANALYSIS PROG)
ALTITUDE (M)
CALCULATED ALTITUDE
1000
800
600
400
200
0
0
200
400
600
800
1000
INTERPOLATED ALTITUDE

Figure 4

COMPARISON OF MEASURED (FROM CONTOUR MAP) AND CALCULATED (FROM TERRAIN ANALYSIS PROG) MAXIMUM SLOPE (0-90 DEG)

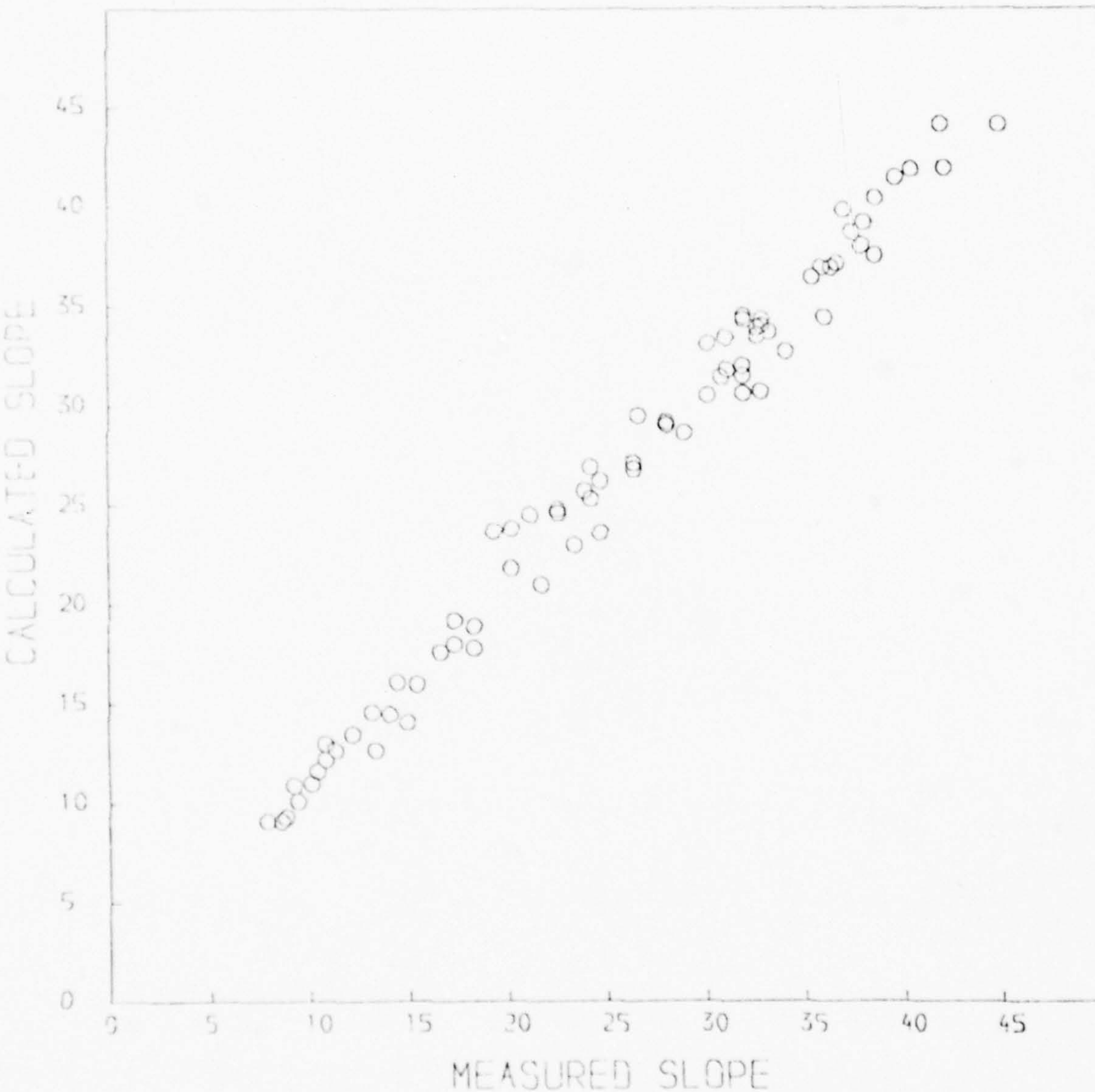
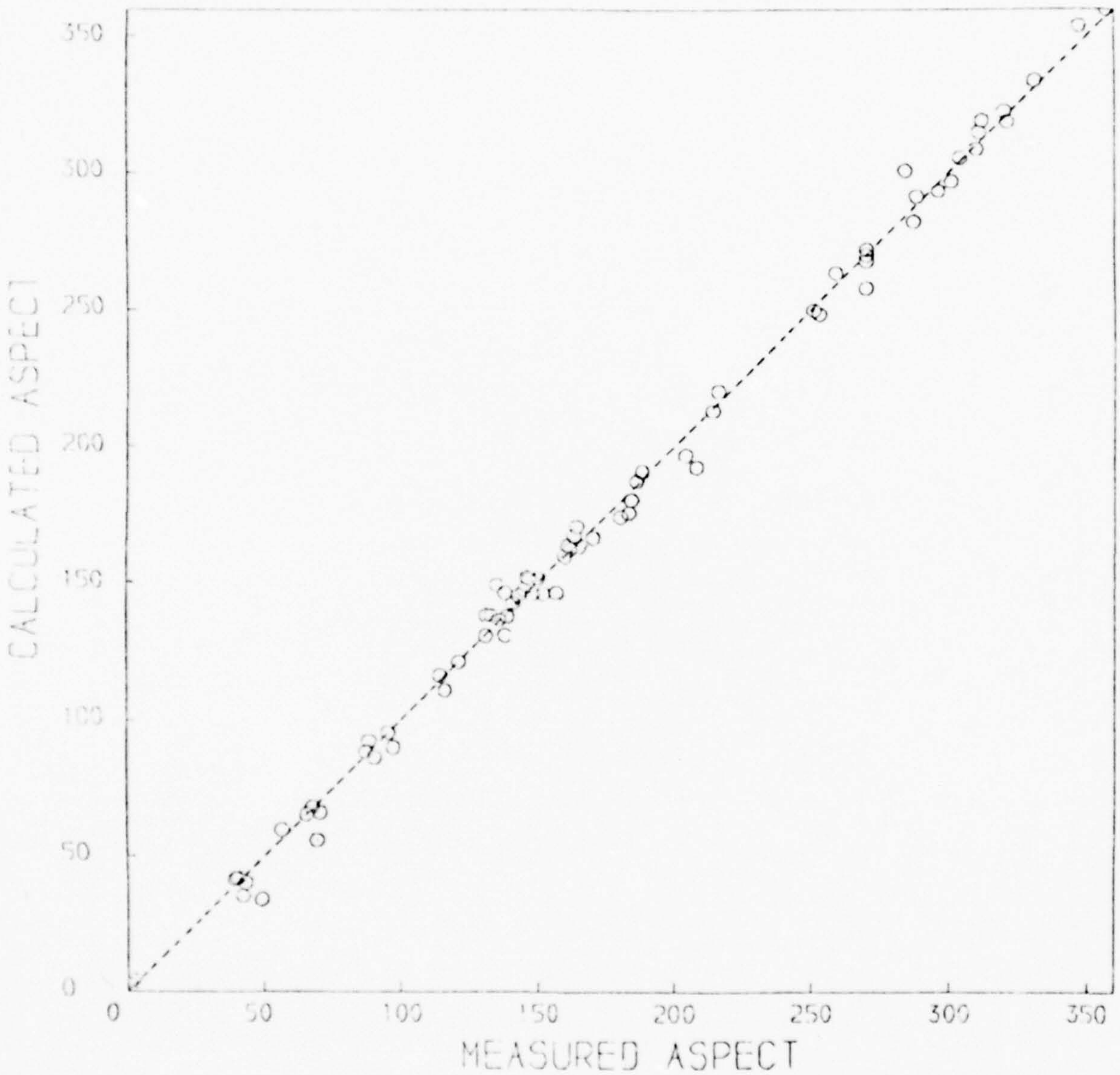


Figure 5

COMPARISON OF MEASURED (FROM CONTOUR MAP) AND CALCULATED (FROM TERRAIN ANALYSIS PROG) ASPECT OF MAXIMUM SLOPE (0-360 DEG)



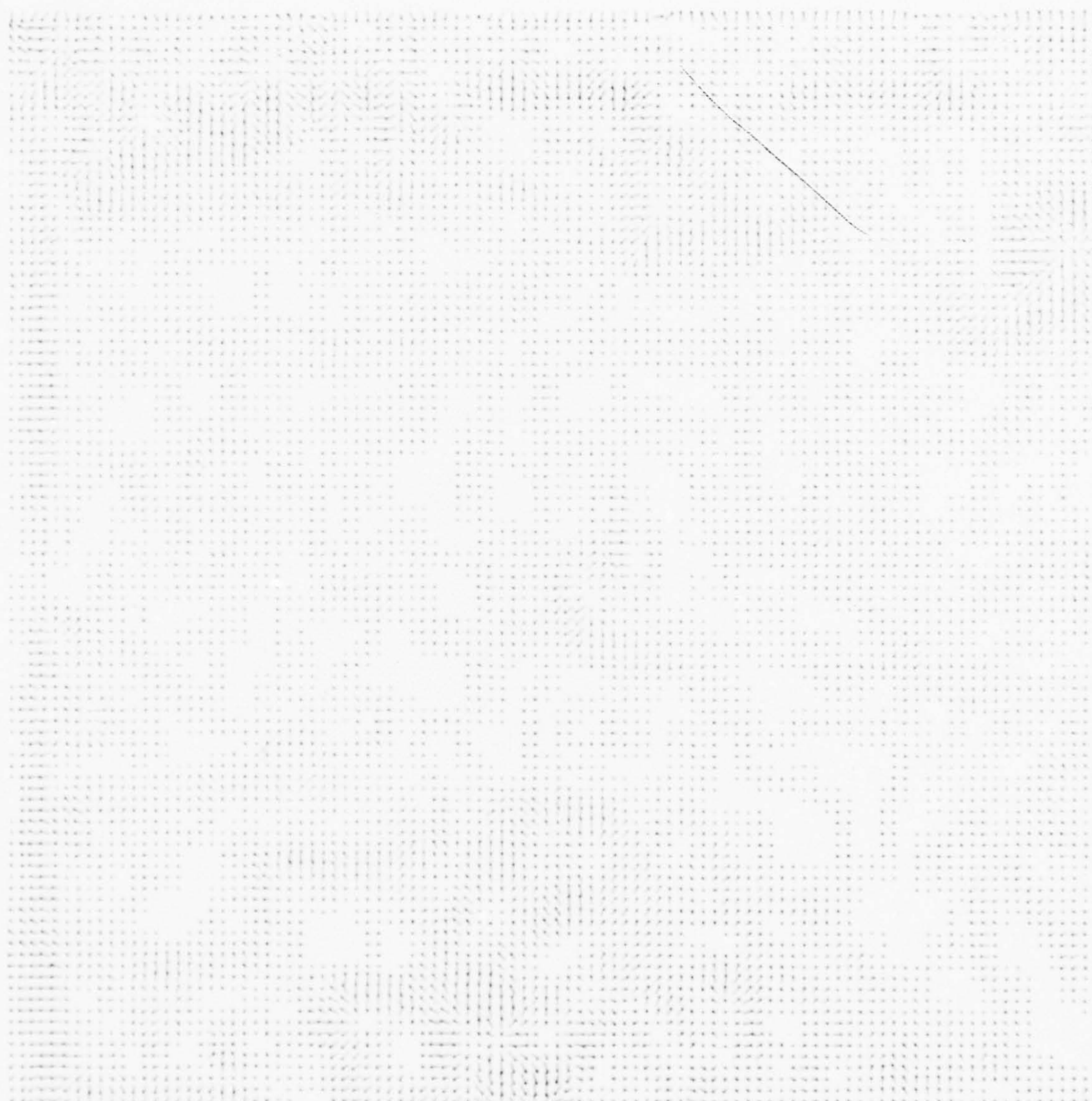
*CILPLOT OUTPUT FOR JOB CLK2 ON SEP 19, 1978 AT 19:12:34

Figure 6 Slope map; Cache 1

CACHE1 SEPT 78

SLOPE VALUES 0.400 1.000 1.600 2.200 2.800

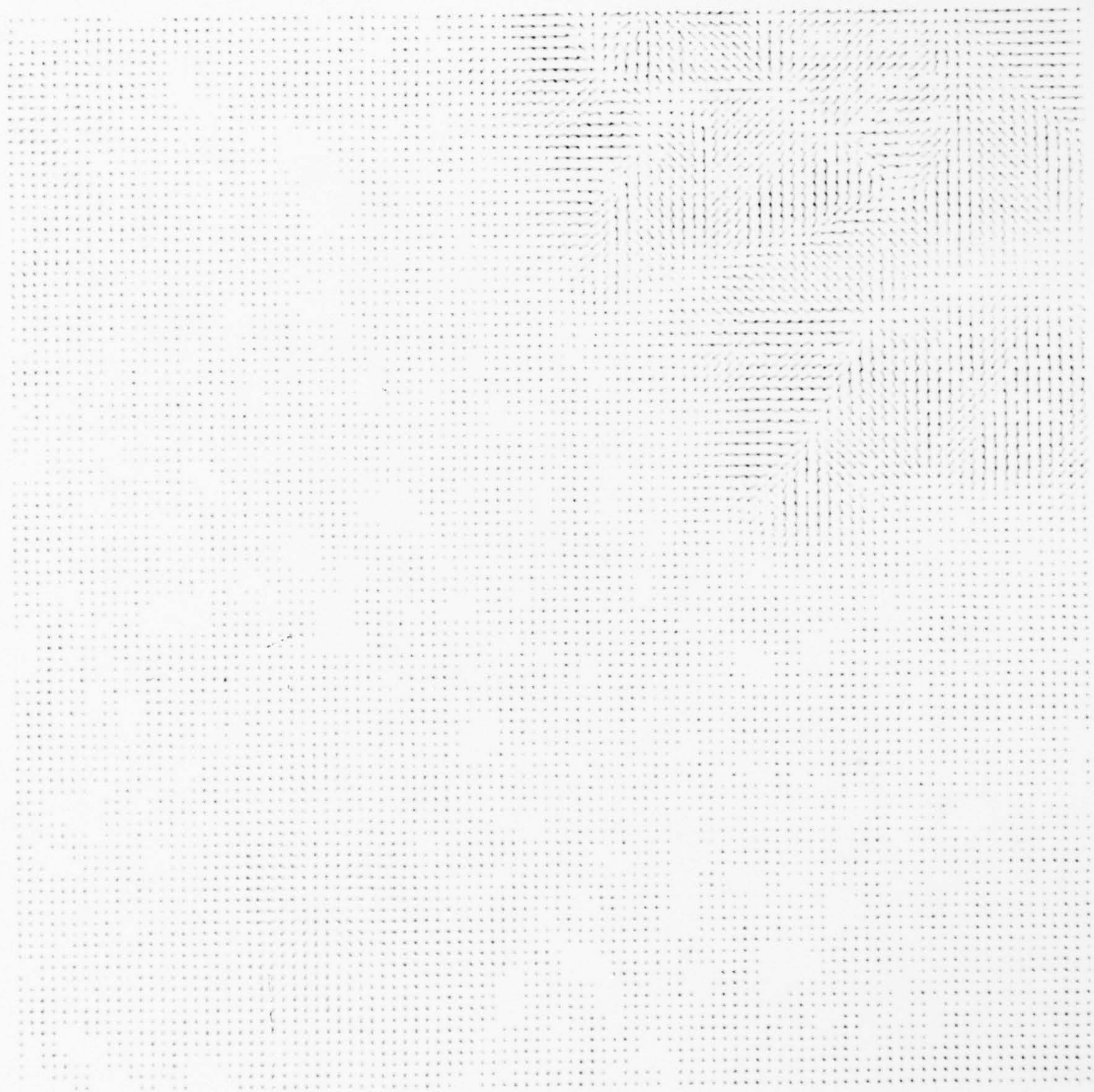
Gradient (degrees)



*CALCOMP PLOT OUTPUT FOR JOB GGM1 ON SEP 21, 1978 AT 18:36:28

Figure 7 Slope map; Cache 2

CACHE2 20 SEPT 1978
SLOPE VALUES -0.000 1.200 3.600 6.000 8.400
Gradient (degrees)



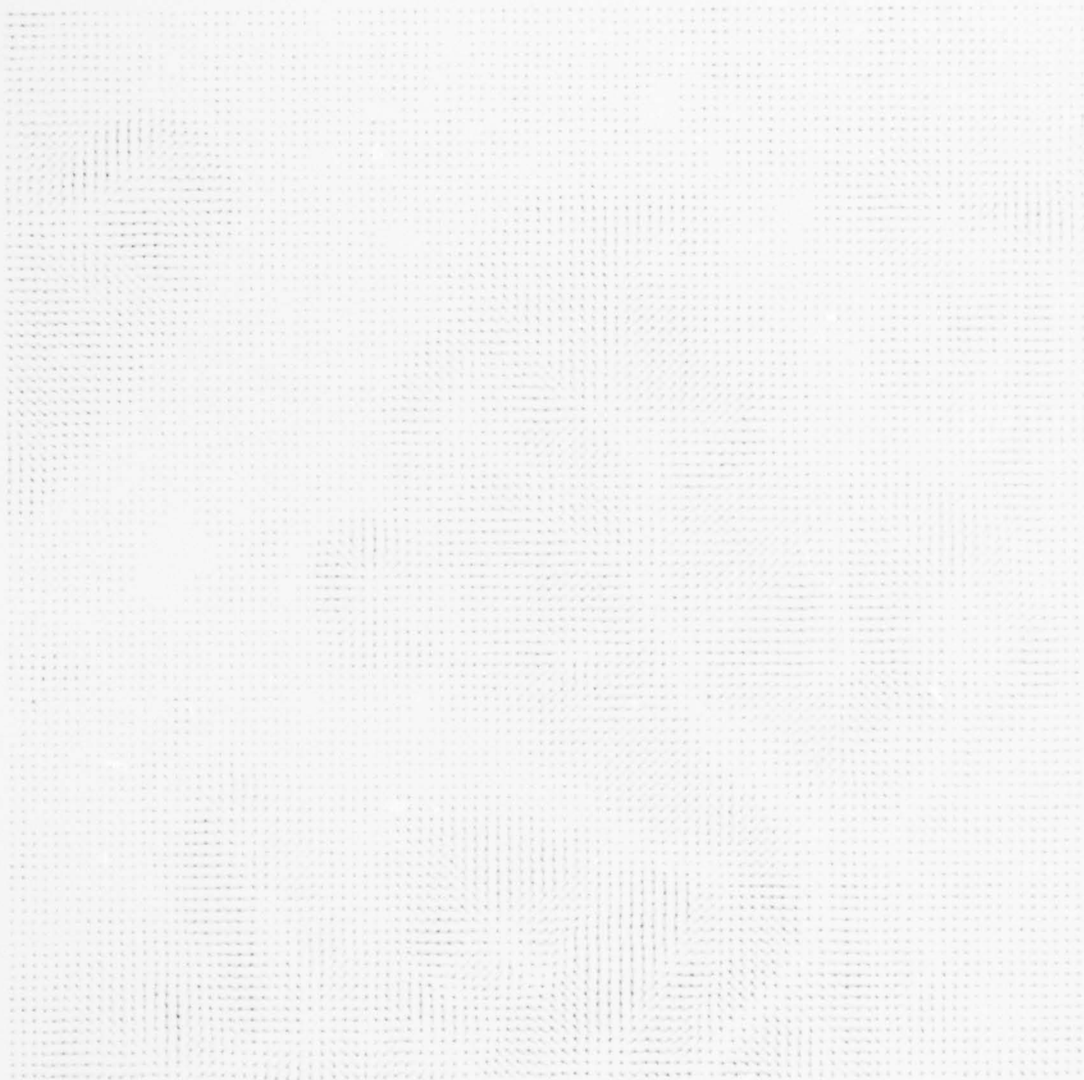
JOB GGM1 ON SEP 21, 1978 AT 18:45:52

Figure 8 Slope map; Cache 3

SEPT 1978

1.200 3.600 6.000 8.400 10.800

Gradient (degrees)

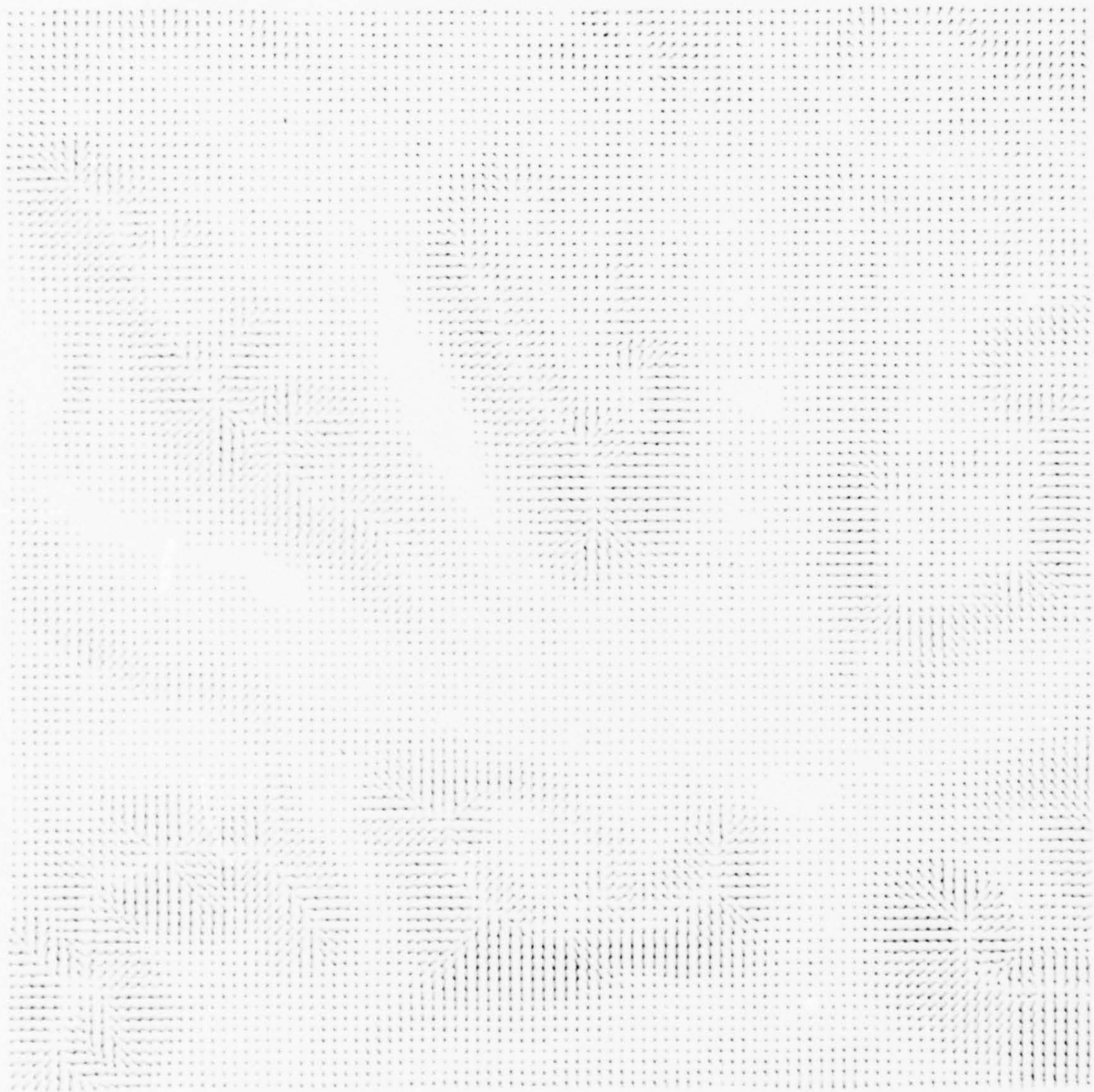


CILPLOT OUTPUT FOR JOB CLK2 ON SEP 16, 1978 AT 05:36:47

Figure 9 Slope map; Torridon

TORRIDON SEPT 78
SLOPE VALUES 1.400 8.100 14.800 21.500 28.200

Gradient (degrees)

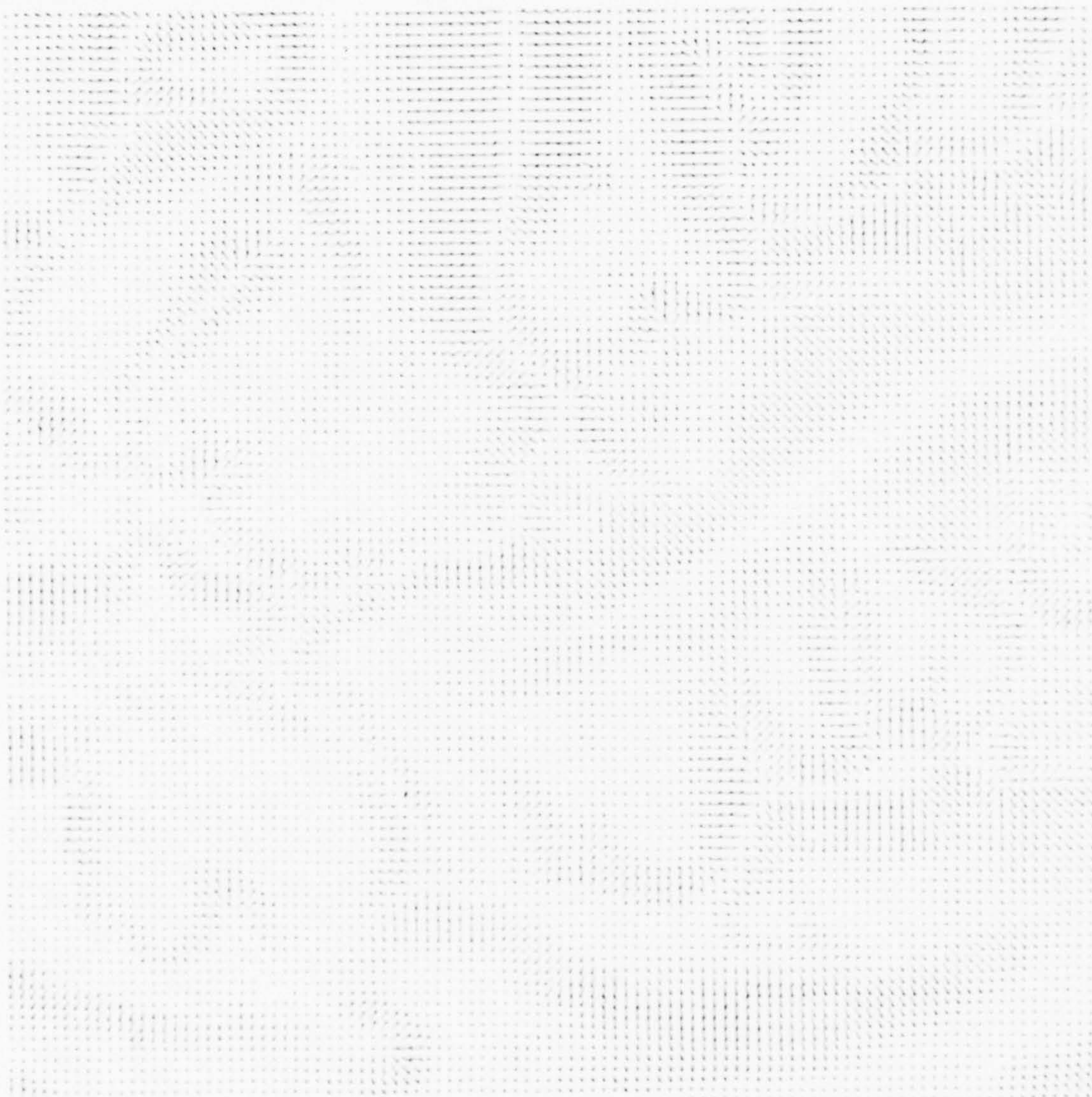


•CILPLOT OUTPUT FOR JOB GGM1 ON SEP 21, 1978 AT 18:57:29

Figure 10 Slope map; Thvera

THVERA, N-CENTRAL ICELAND
SLOPE VALUES 8.800 15.100 21.400 27.700 34.000

Gradient (degrees)

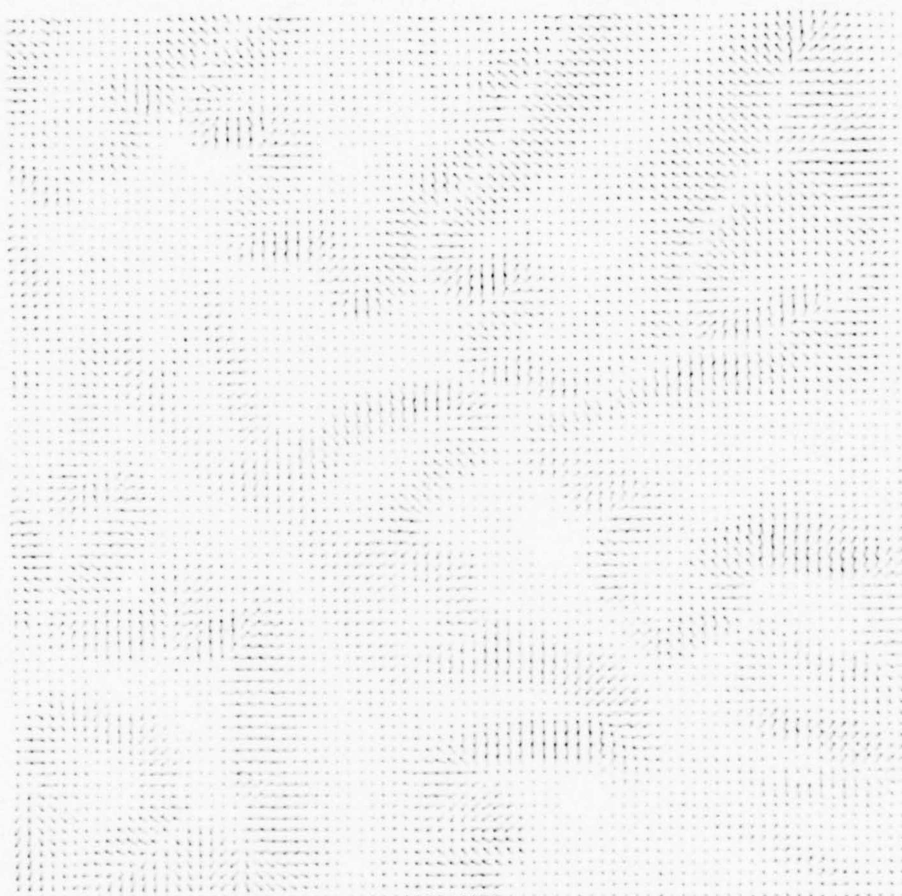


*CILPLOT OUTPUT FOR JOB CLK2 ON SEP 15, 1978 AT 18:24:00

Figure 11 Slope map; Nupur

NUPUR 7 SEPT 78
SLOPE VALUES 6.900 14.400 21.900 29.400 36.900

Gradient (degrees)



*CILPLOT OUTPUT FOR JOB GGM1 ON SEP 15, 1978 AT 19:31:19

Figure 12 Slope map; Ferro

FERRO BASIN, N. CALABRIA 13 SEPT 1978
SLOPE VALUES 7.100 10.100 13.100 16.100 19.100

Gradient (degrees)

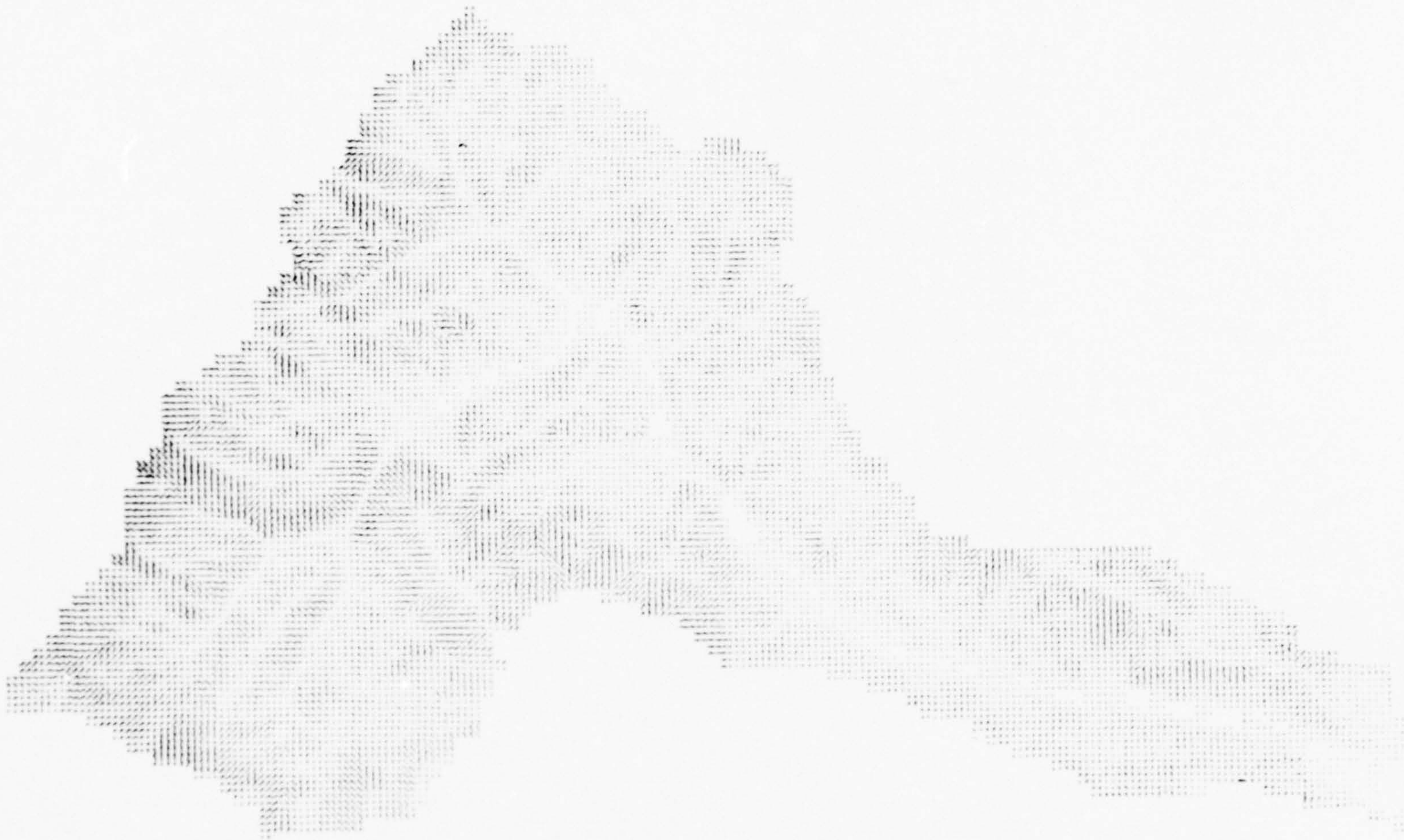
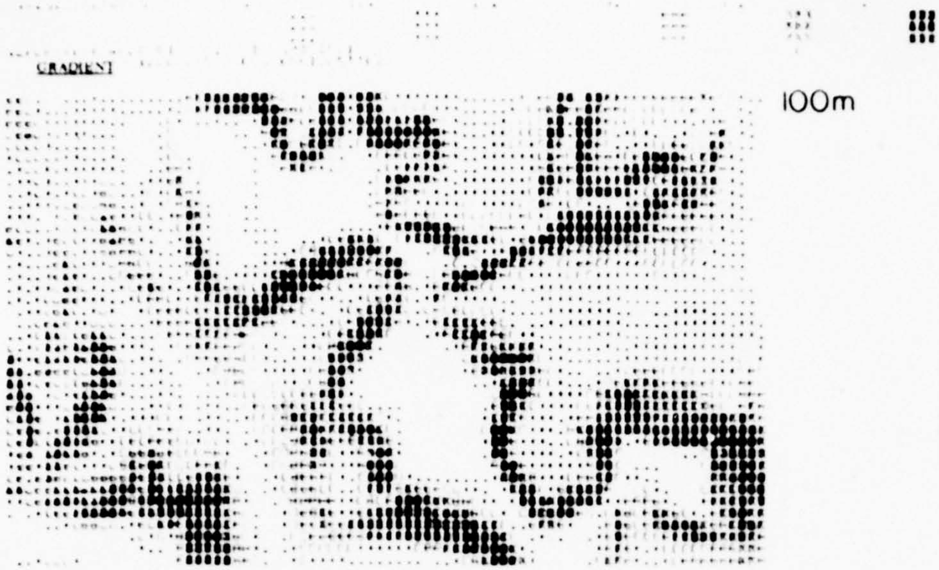


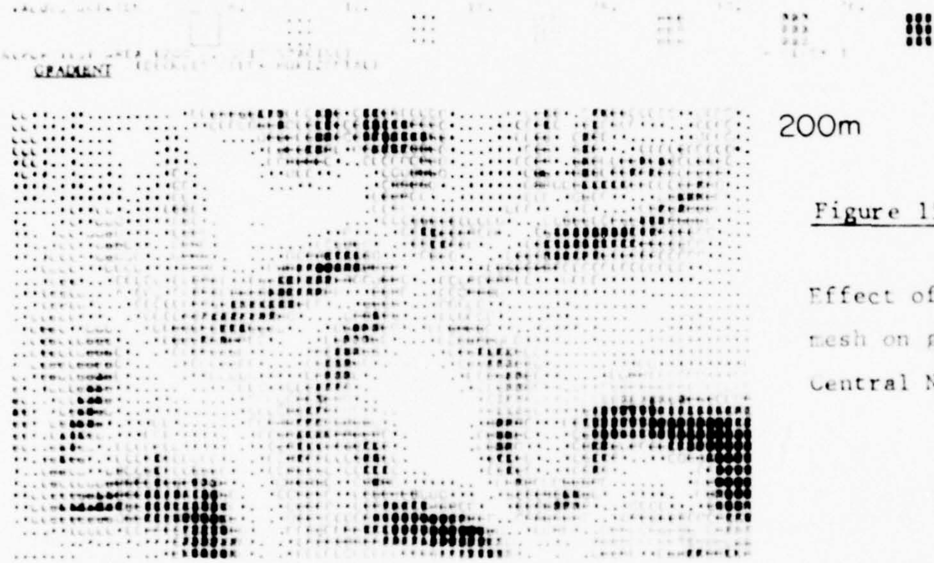
Figure 13 The Nupur area; incidence of grid.

Contoured by GPCP; interval 20m. 'Central Nupur' is defined by the heavy line.

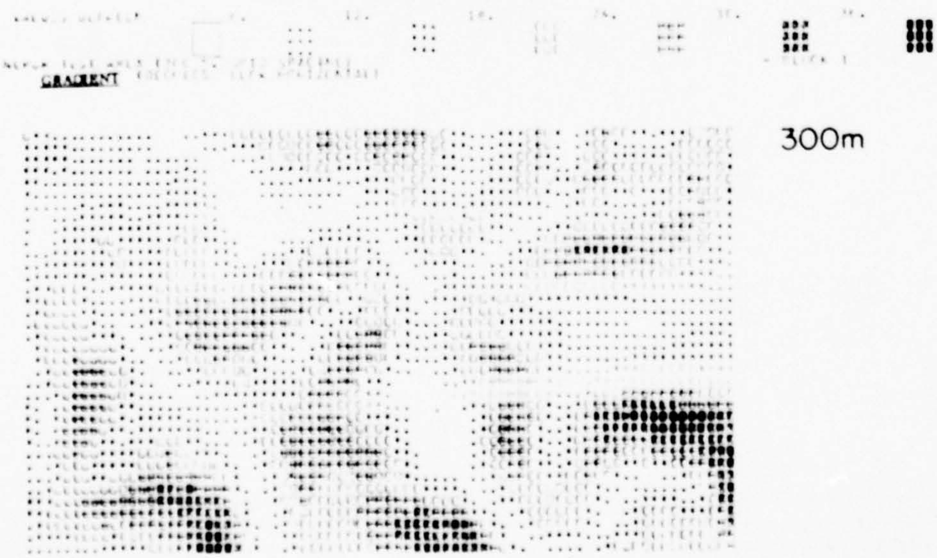
Figure 14 Effect of 1-2-1 weighted smoothing on aspect of slopes steeper than 20 degrees, in Central Nupur. (Top) smoothed once, (middle) smoothed twice, (bottom) smoothed three times .



100m



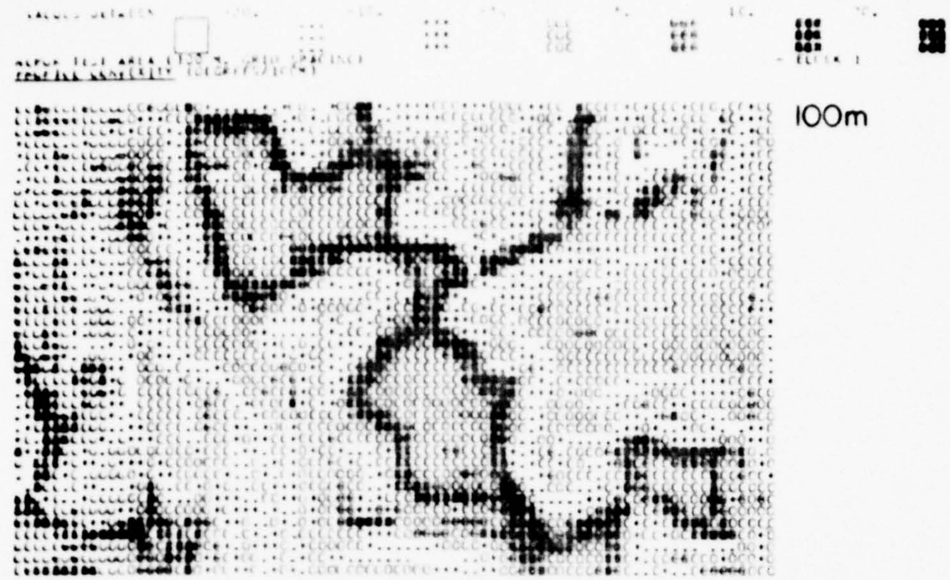
200m

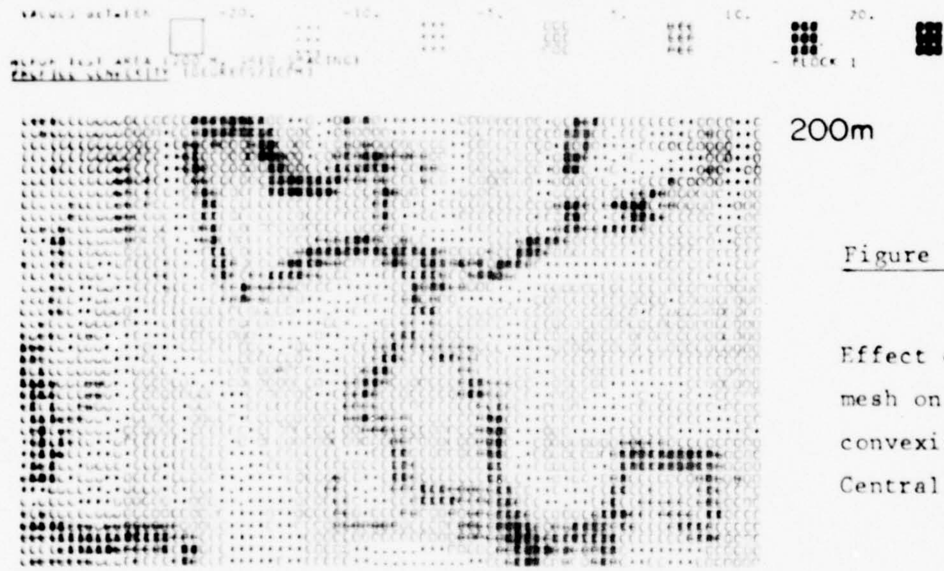


300m

Figure 15

Effect of grid mesh on gradient, Central Nupur .





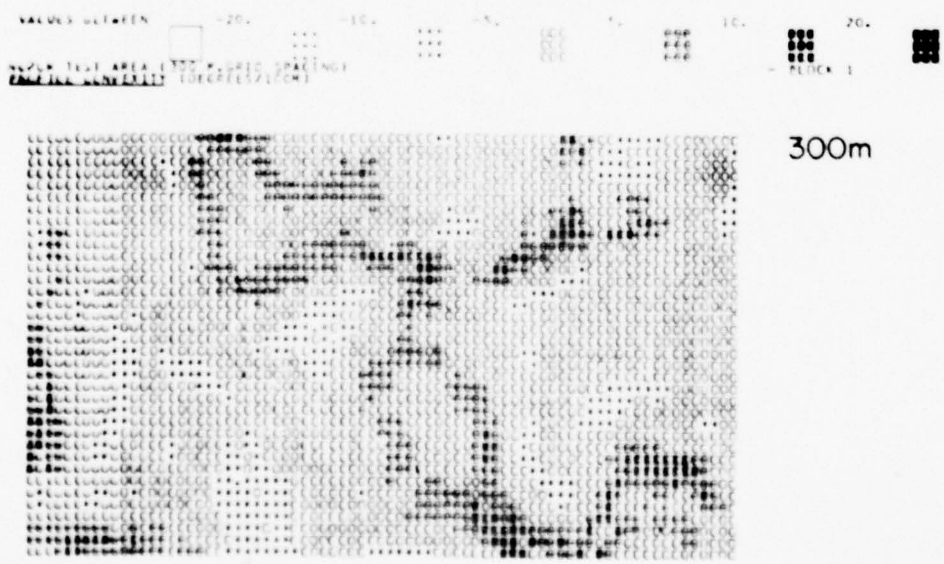


Figure 16

Effect of grid mesh on profile convexity, Central Nupur

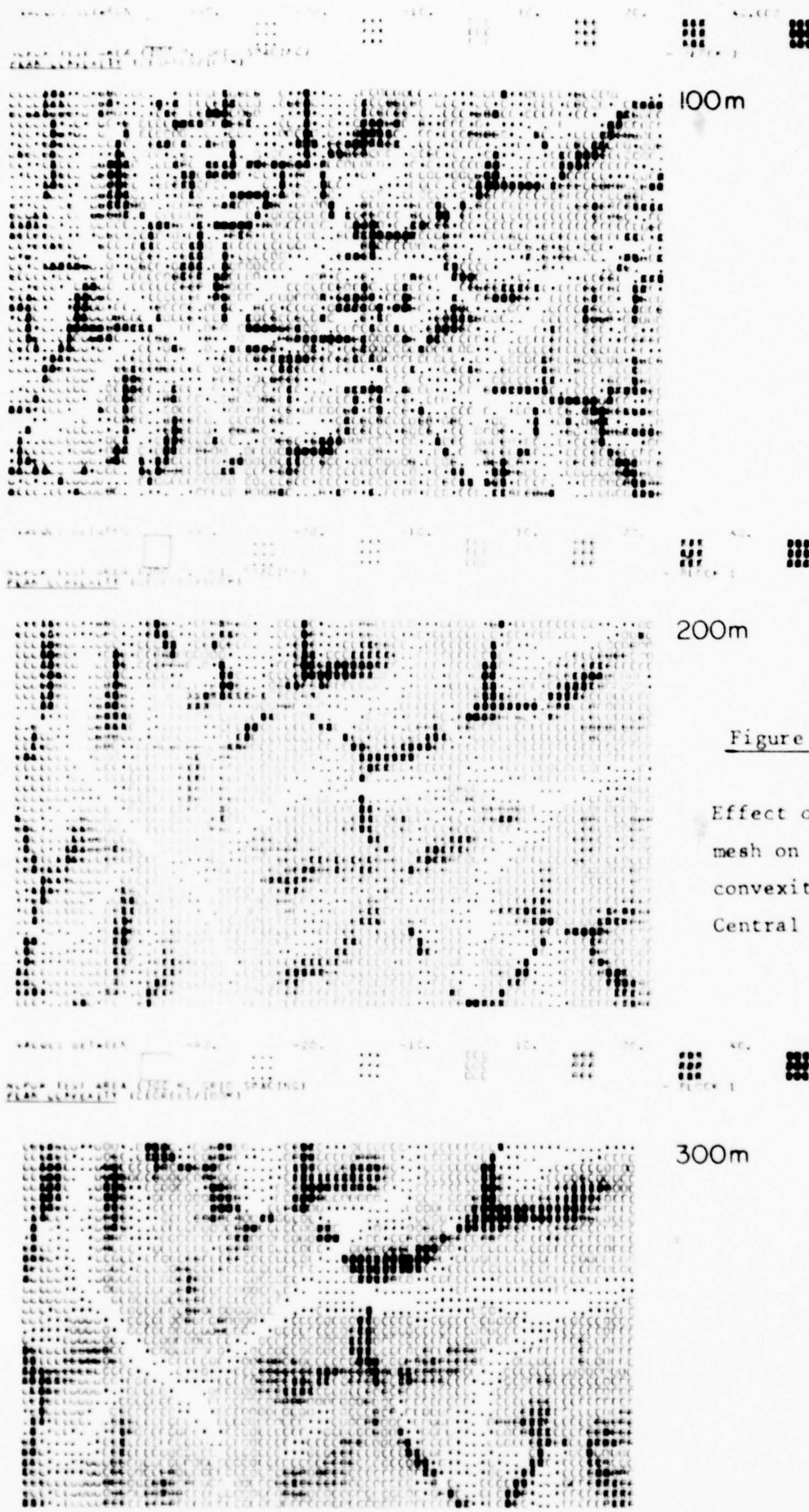


Figure 17

Effect of grid mesh on plan convexity, Central Nupur .

Figure 18 Ferro;

histogram of altitude .

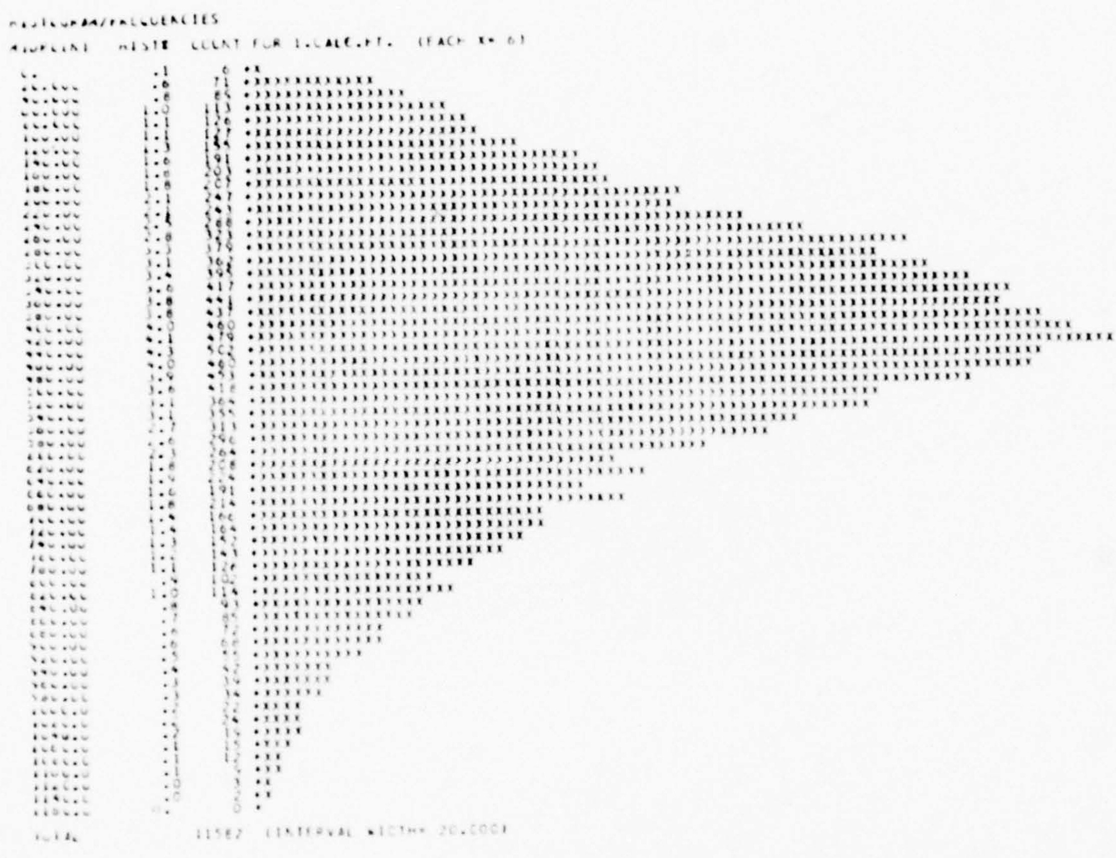


Figure 19 Ferro,

histogram of aspect .

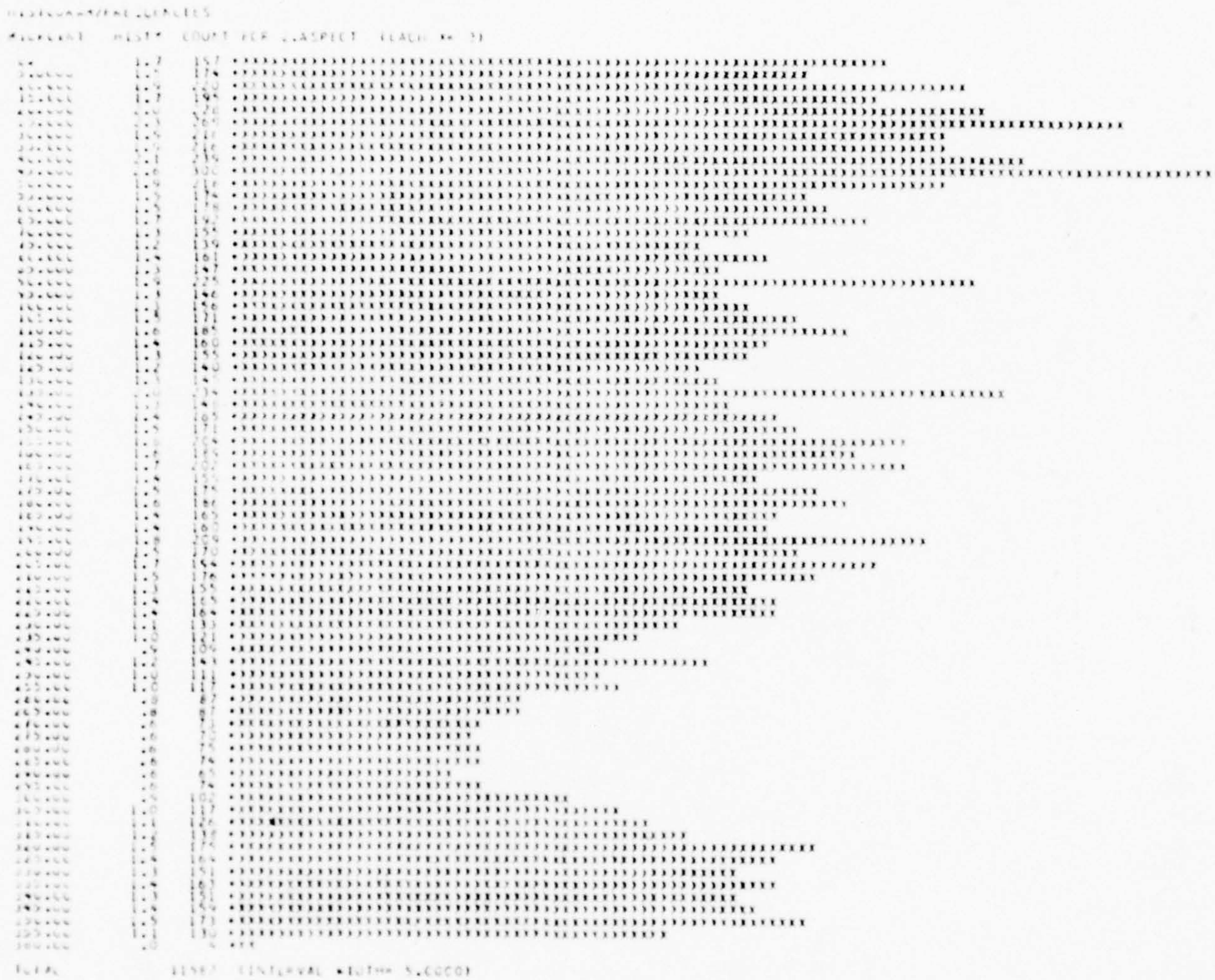


Figure 20 Ferro;
histogram of gradient .

HISTOGRAM/FREQUENCIES

MIDPOINT	HIST%	COUNT FOR 3.GRADIENT (EACH X= 12)
0.	0.	0
1.0000	1.1	124
2.0000	1.2	139
3.0000	2.3	261
4.0000	1.5	173
5.0000	1.8	209
6.0000	2.5	287
7.0000	2.5	289
8.0000	3.7	425
9.0000	5.0	584
10.000	5.9	687
11.000	9.5	1105
12.000	8.0	929
13.000	9.5	1100
14.000	7.5	870
15.000	8.0	928
16.000	6.2	716
17.000	5.7	663
18.000	4.8	559
19.000	3.5	404
20.000	2.8	325
21.000	2.2	254
22.000	1.7	194
23.000	1.0	114
24.000	.8	88
25.000	.4	50
26.000	.3	35
27.000	.3	29
28.000	.1	10
29.000	.1	15
30.000	.1	7
31.000	0.	0
32.000	.0	3
33.000	.0	2
34.000	.0	2
35.000	.0	2

TOTAL 11582 (INTERVAL WIDTH= 1.0000)

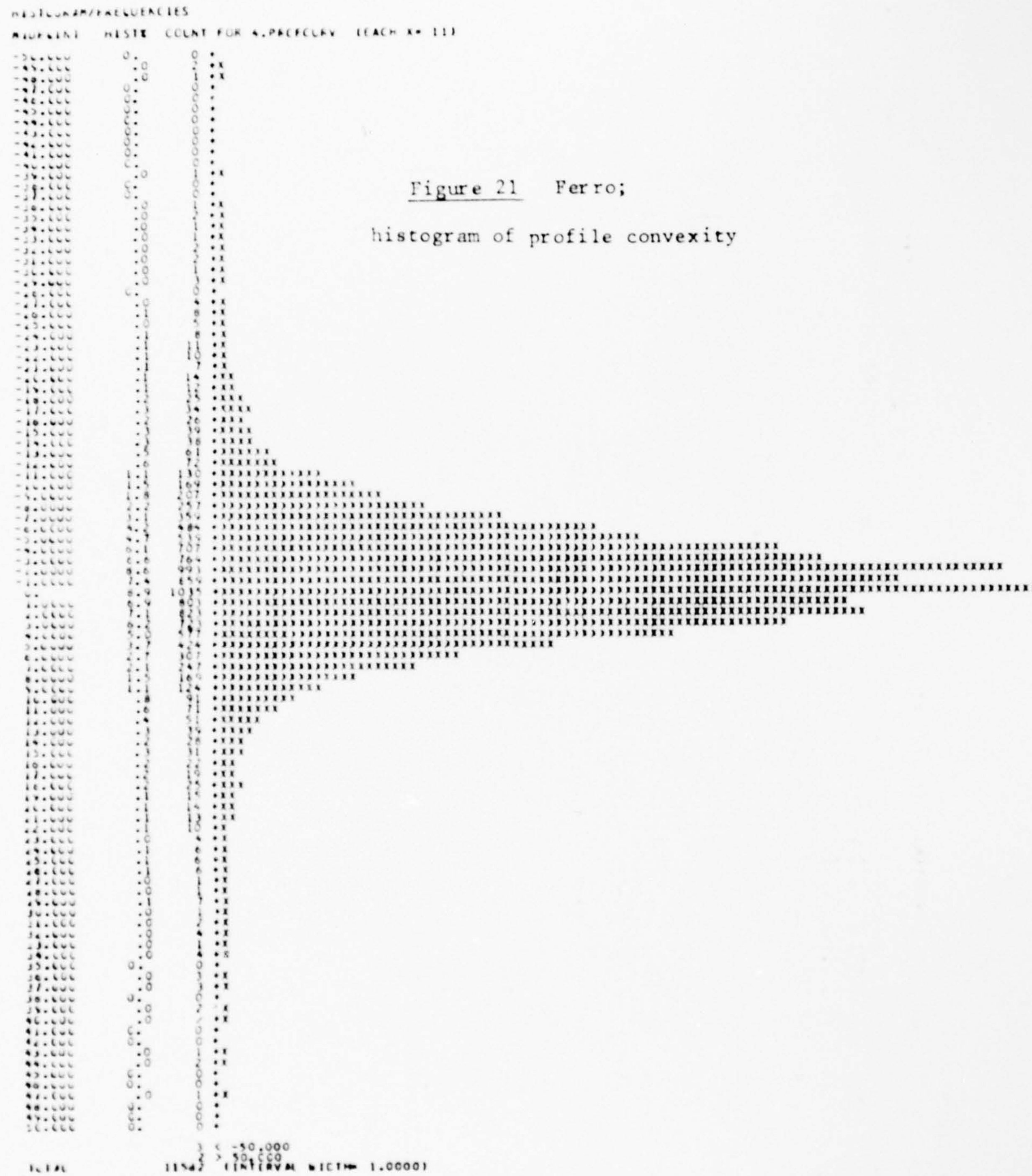


Figure 21 Ferro;

histogram of profile convexity

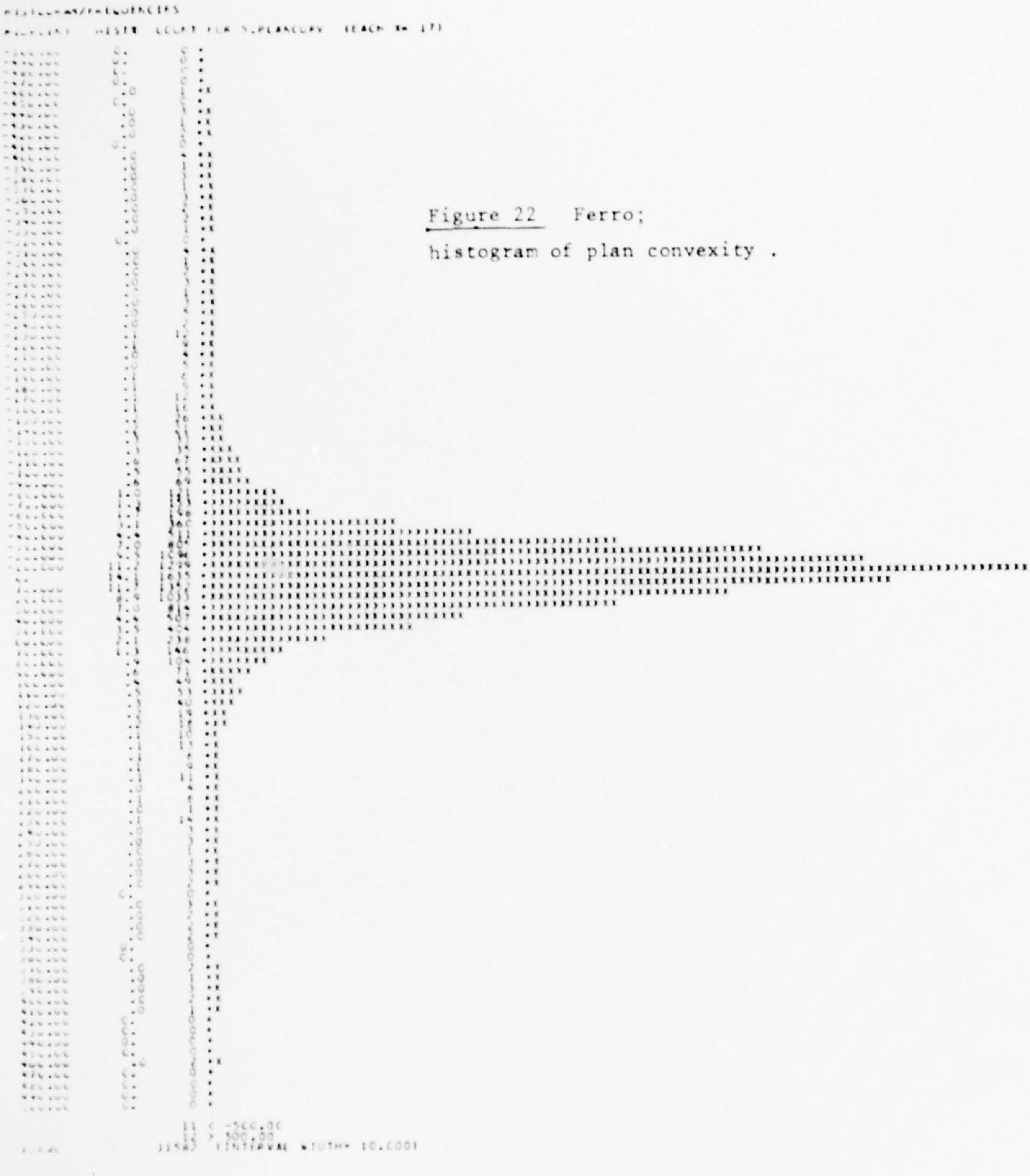


Figure 22 Ferro;
histogram of plan convexity .

Figure 23 Upper and central Ferro; map of altitude.

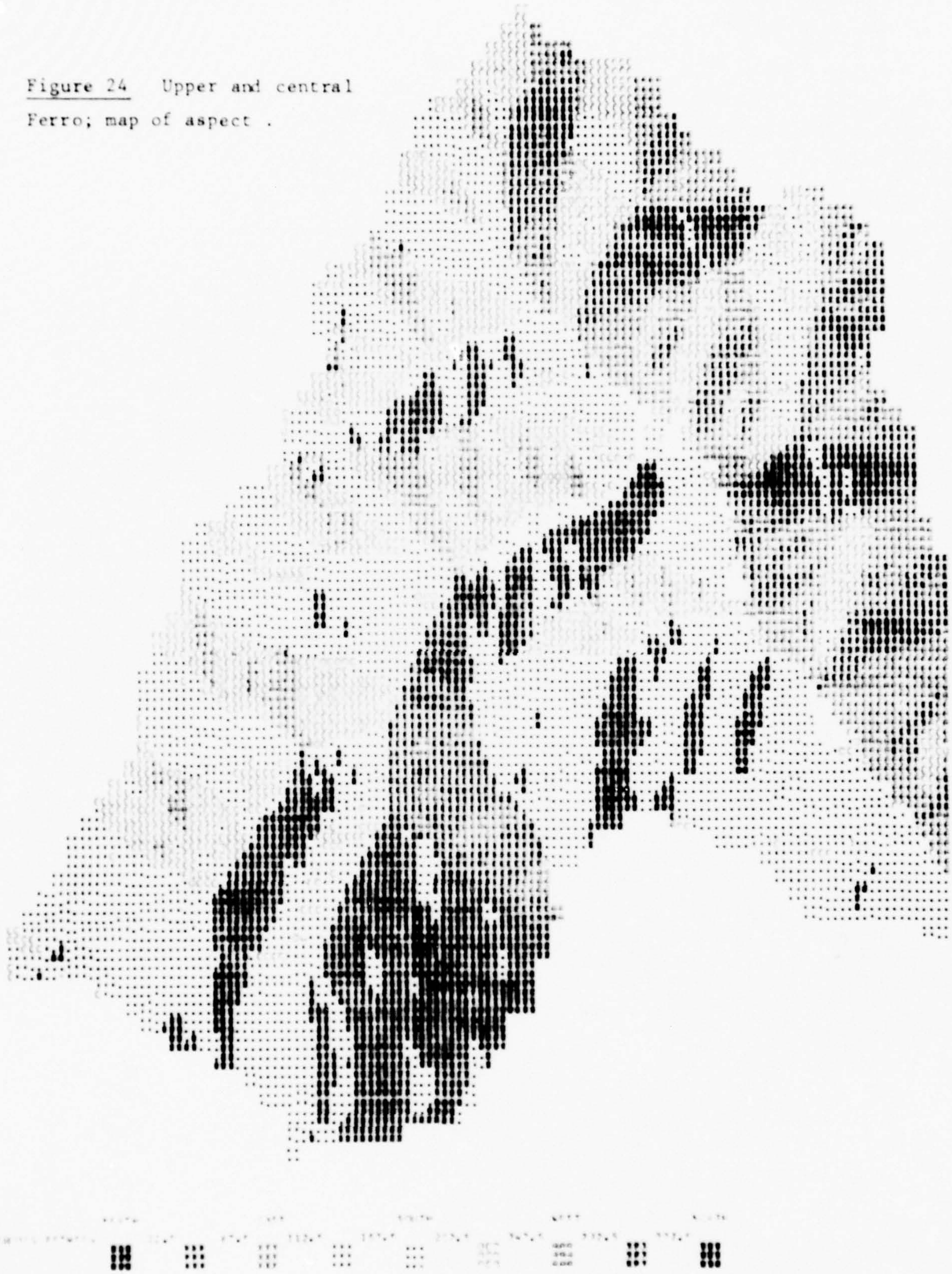


Figure 24 Upper and central Ferro; map of aspect .

Figure 25 Upper and central Ferro; map of gradient .

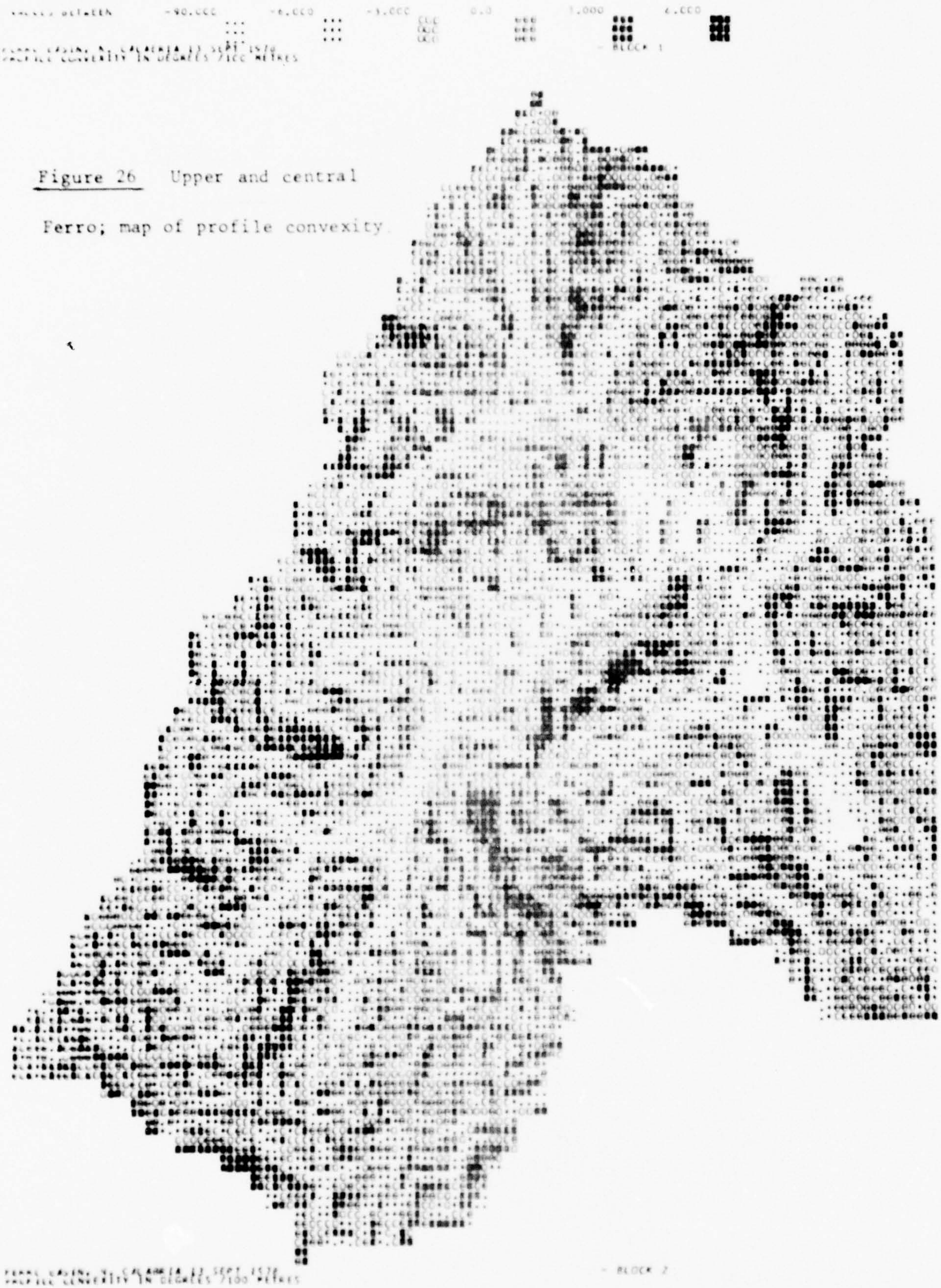


Figure 26 Upper and central Ferro; map of profile convexity

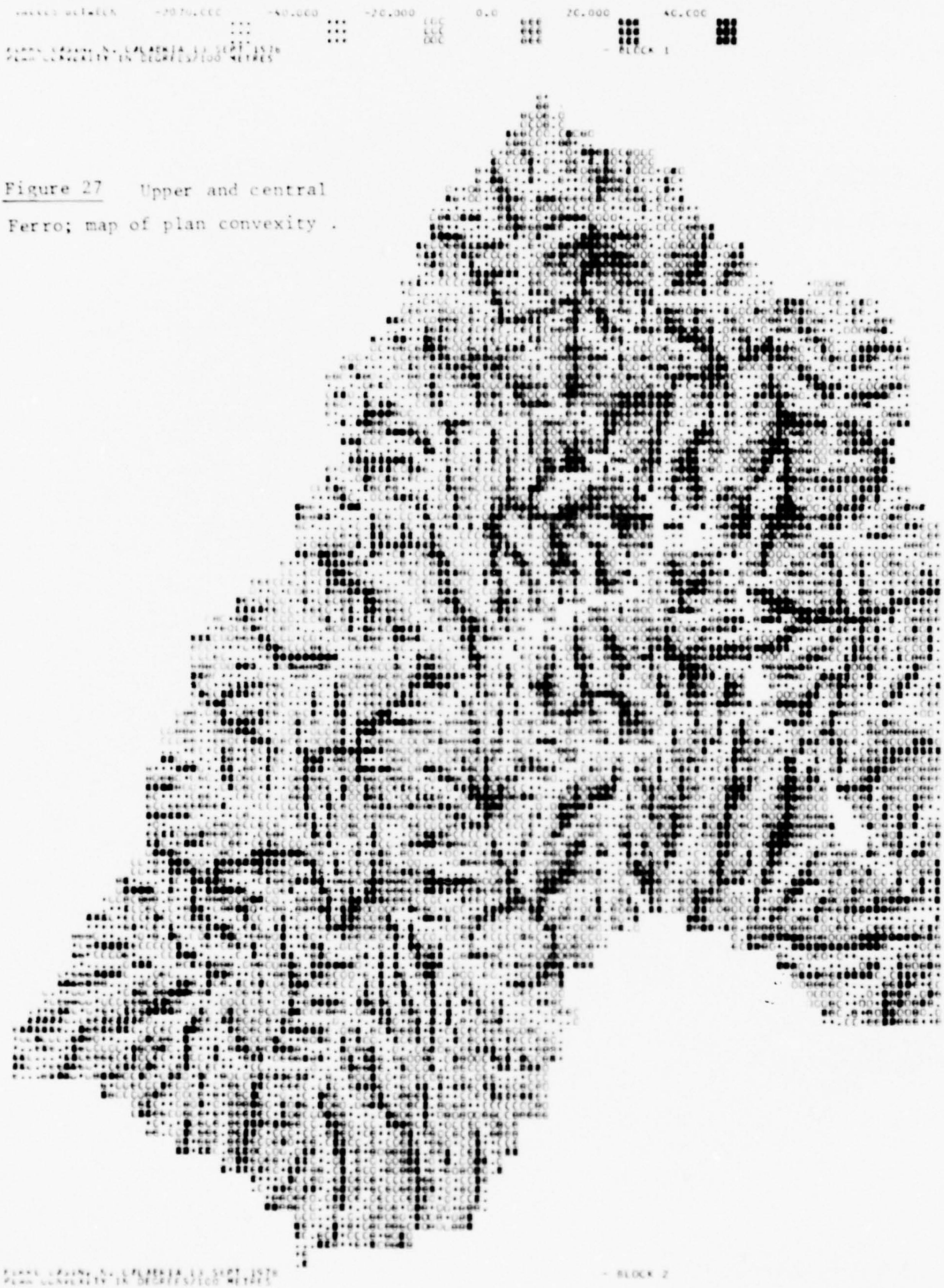


Figure 27 Upper and central Ferro; map of plan convexity .

FERRO BASIN, N. CALABRIA 13 SEPT 1978 09/15/78

FILE NONAME (CREATION DATE = 09/15/78)
SCATTERGRAM OF (DOWN) F CALCULATED HEIGHT USING FITTED Q (ACROSS) SLOPE GRADIENT OF MAX SLOPE IN DEG.

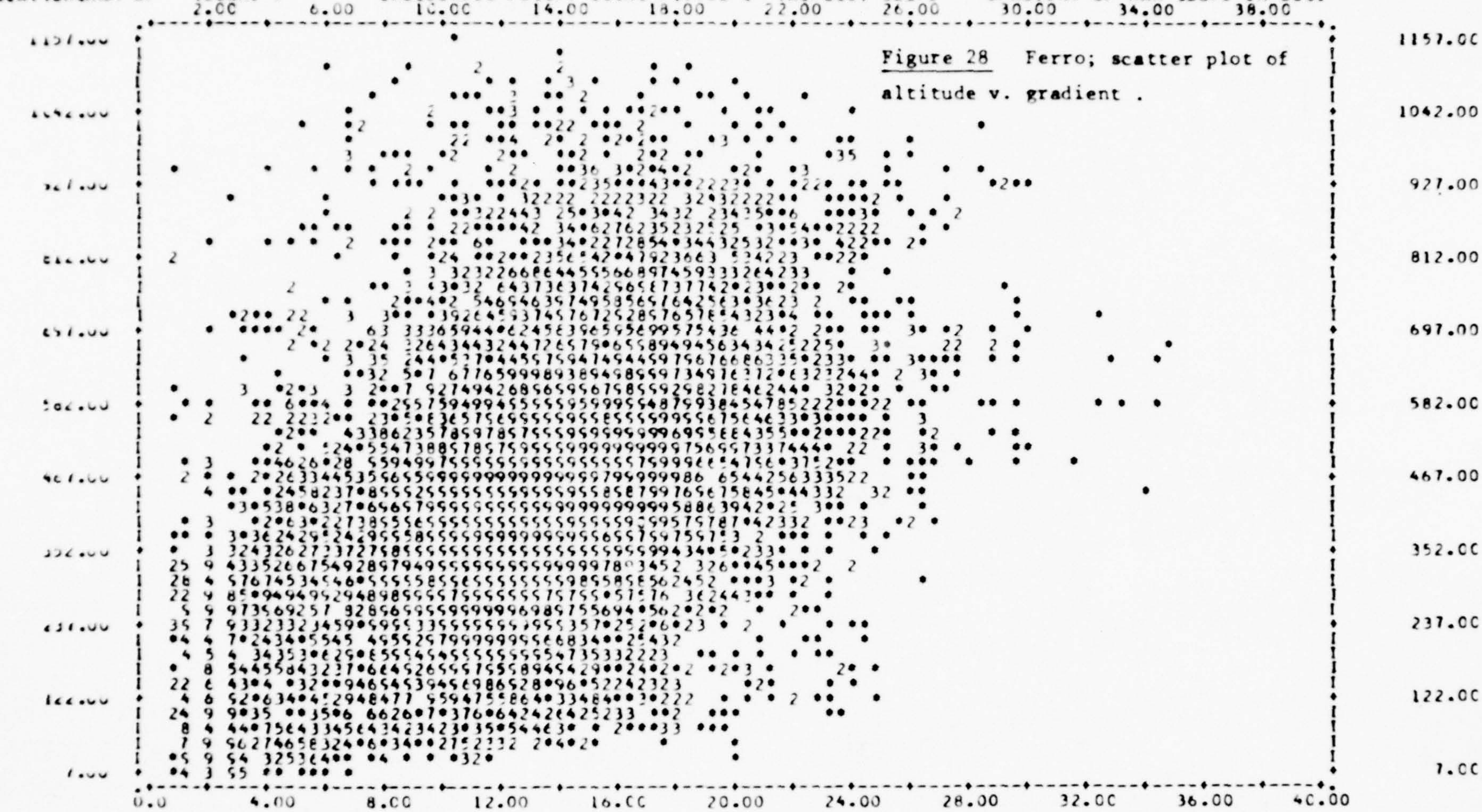


Figure 28 Ferro; scatter plot of altitude v. gradient .

FERRO CASINO, N. CALABRIA 13 SEPT 1978 09/15/78

FILE NONAME (CREATION DATE = 09/15/78)
SCATTERGRAM OF (DOWN) PROFC PROFILE CONVEXITY ON MAX SLOPE-D (ACROSS) PLANC PLAN CONVEXITY IN DEG:100M

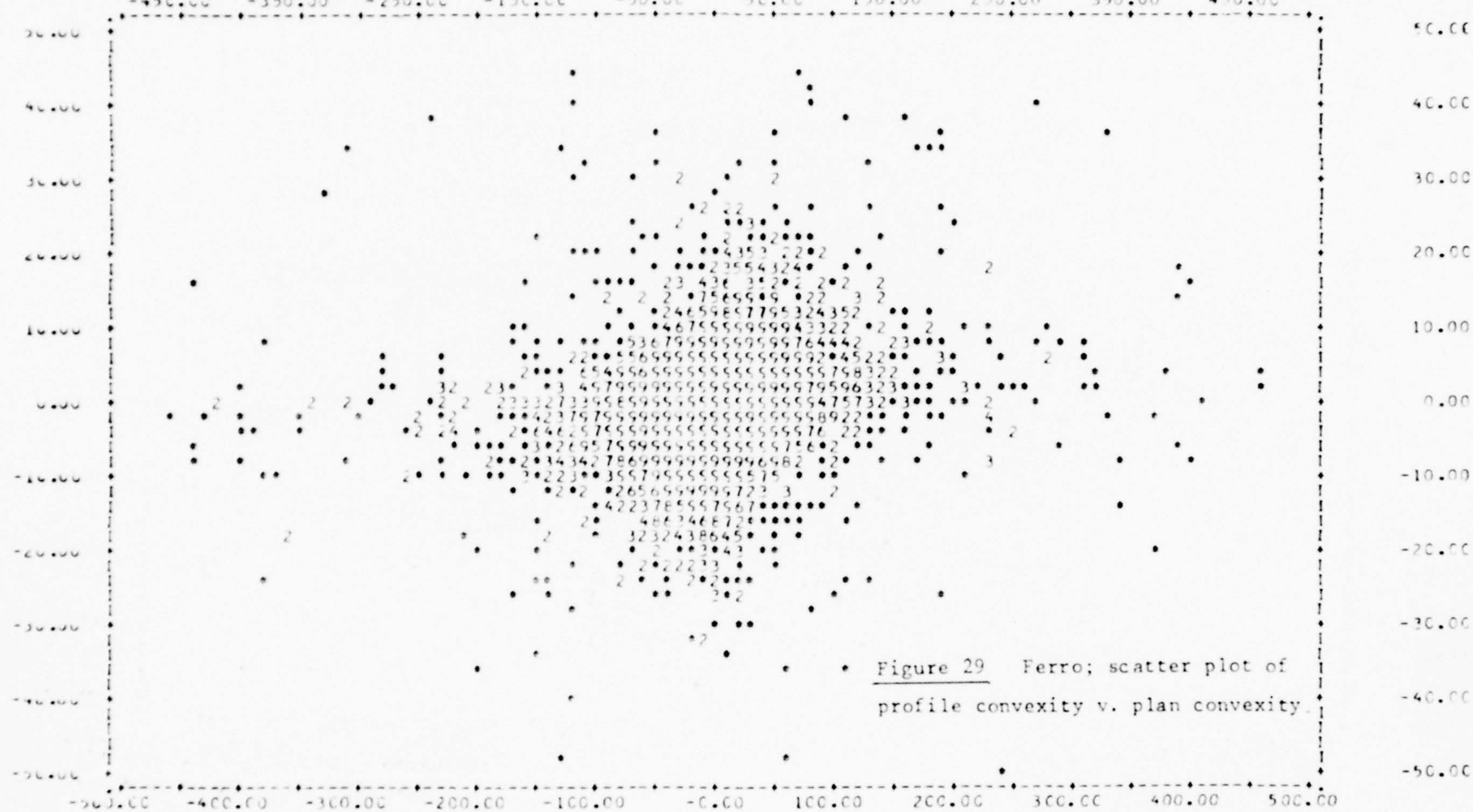


Figure 29 Ferro; scatter plot of profile convexity v. plan convexity.

FERRO BASIN, N. CALABRIA 13 SEPT 1978 05/15/78

FILE NONAME (CREATION DATE = 05/15/78)
SCATTERGRAM OF (DOWN) SLOPE GRADIENT OF MAX SLOPE IN DEG. (ACROSS) PLANC PLAN CONVEXITY IN DEG:100M

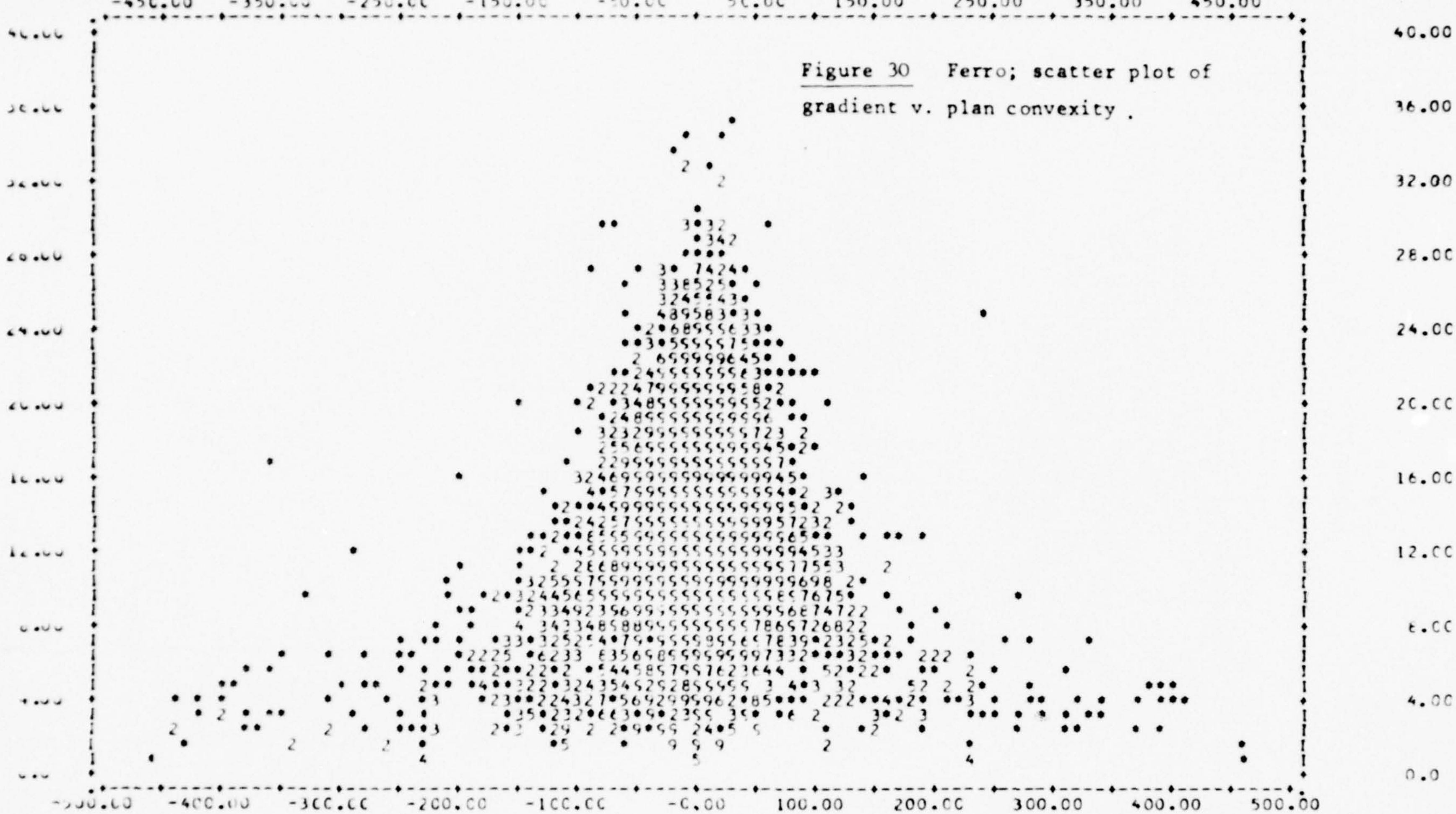


Figure 30 Ferro; scatter plot of gradient v. plan convexity.

Figure 31 Cache 2;
histogram of altitude .

Figure 32 Cache 2;

histogram of aspect .

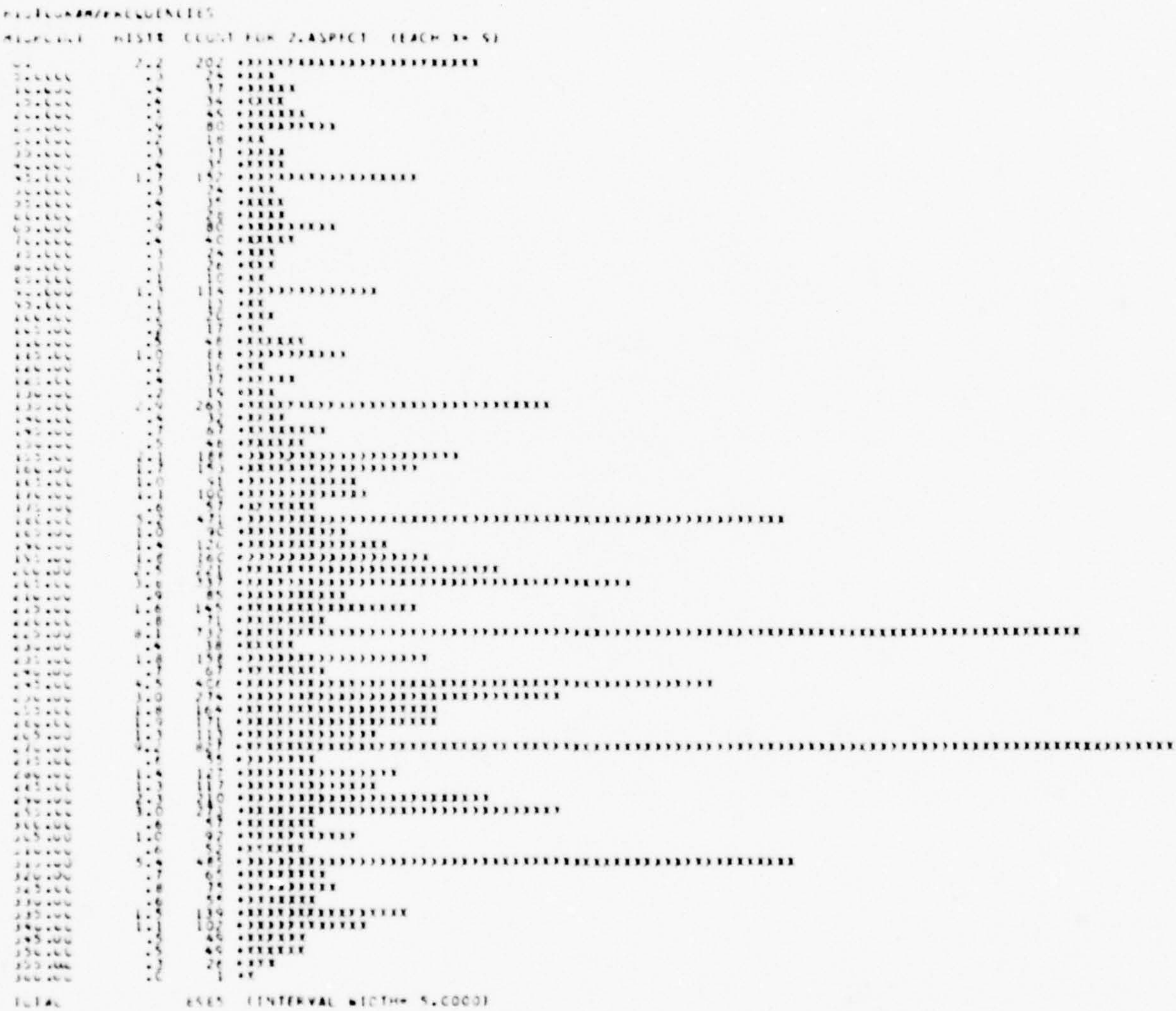


Figure 33 Cache 2;

histogram of gradient .

```
HISTOGRAM/FREQUENCIES
MIDPOINT   HIST%  COUNT FOR 3.GRADIENT  (EACH X= 37)
  0.          1.5    132 *XXXX
  1.00000    41.6   3644 *XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  2.00000    23.7   2128 *XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  3.00000     7.6    658 *XXXXXXXXXXXXXXXXXX
  4.00000     4.2    375 *XXXXXXXXXXX
  5.00000     2.6    232 *XXXXXXX
  6.00000     1.9    172 *XXXXX
  7.00000     1.9    170 *XXXXX
  8.00000     2.0    176 *XXXXX
  9.00000     2.3    205 *XXXXXX
 10.00000     2.6    238 *XXXXXXX
 11.00000     2.3    205 *XXXXXX
 12.00000     2.1    190 *XXXXXX
 13.00000     1.4    122 *XXXX
 14.00000     1.0     89 *XXX
 15.00000     1.0     87 *XXX
 16.00000      .5     47 *XX
 17.00000      .3     27 *X
 18.00000      .2     13 *X
 19.00000      .1     11 *X
 20.00000      .1      7 *X
 21.00000      .0      2 *X
 22.00000      .0      3 *X
 23.00000      .0      3 *X
TOTAL                 8585  (INTERVAL WIDTH= 1.0000)
```

Figure 34 Cache 2;
histogram of profile convexity .

Figure 35 Cache 2;
histogram of plan convexity .

Figure 36 Cache 2; map of altitude .

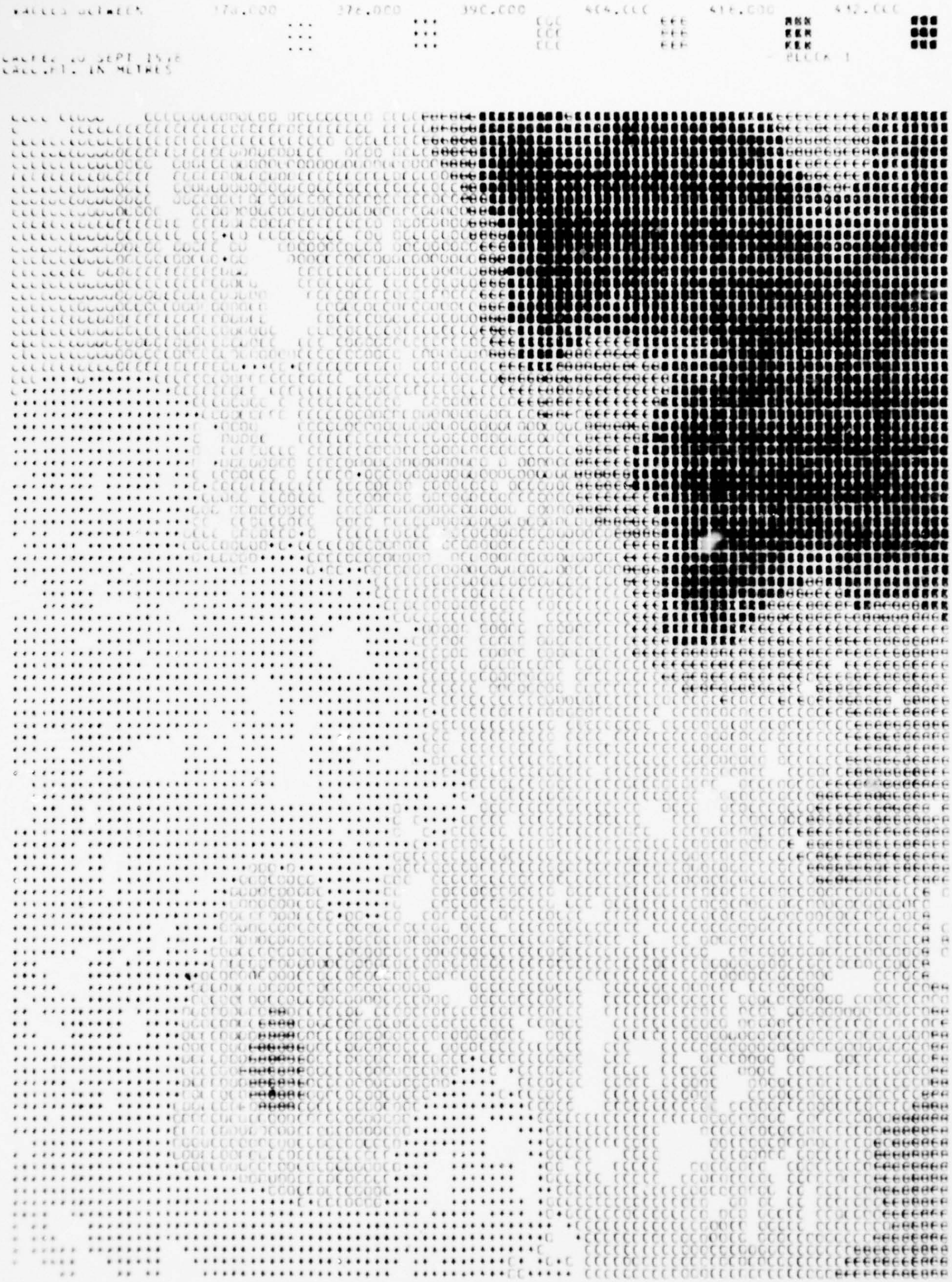


Figure 37 Cache 2; map of aspect .

Figure 38 Cache 2; map of gradient .

Figure 39 Cache 2; map of profile convexity .

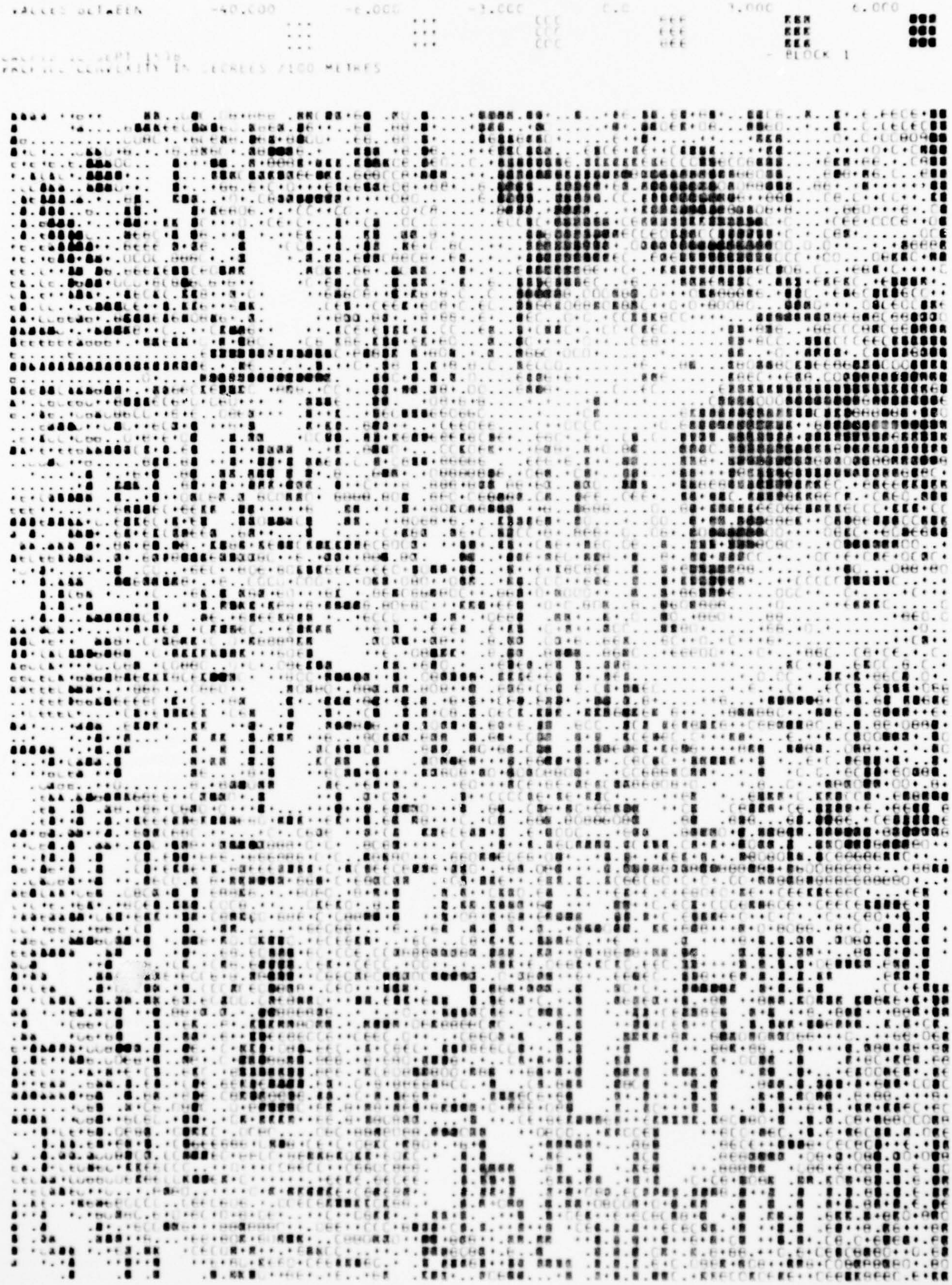


Figure 40 Cache 2; map of plan convexity .

CACHE2 20 SEPT 1978 10/02/78

FILE NONAME (CREATION DATE = 10/02/78)
SCATTERGRAM OF (DOWN) PLANC PLAN CONVEXITY IN DEG:100M (ACROSS) F CALCULATED HEIGHT USING FITTED

Figure 41 Cache 2; scatter plot of plan convexity v. altitude .

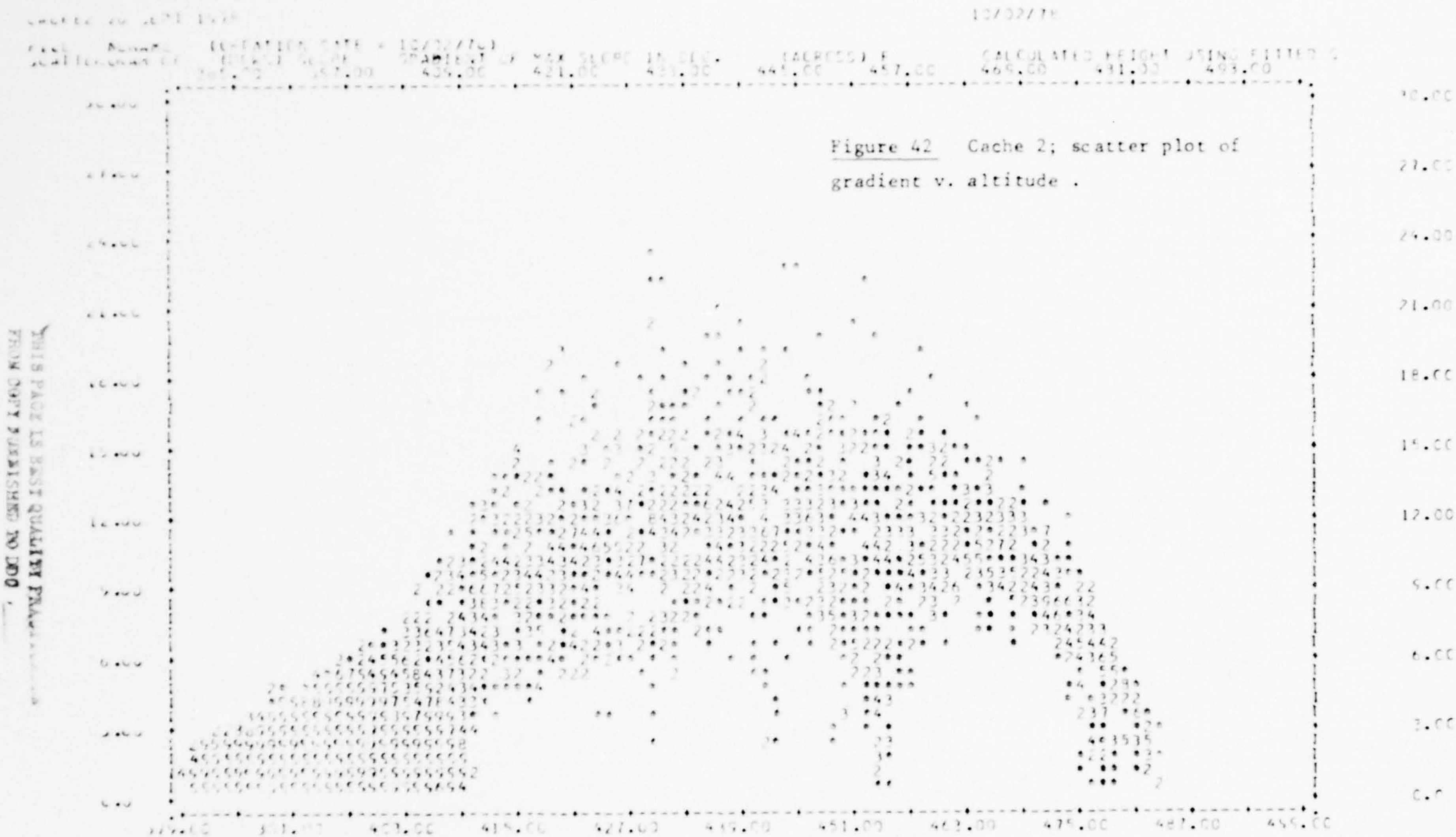


Figure 42 Cache 2; scatter plot of gradient v. altitude .

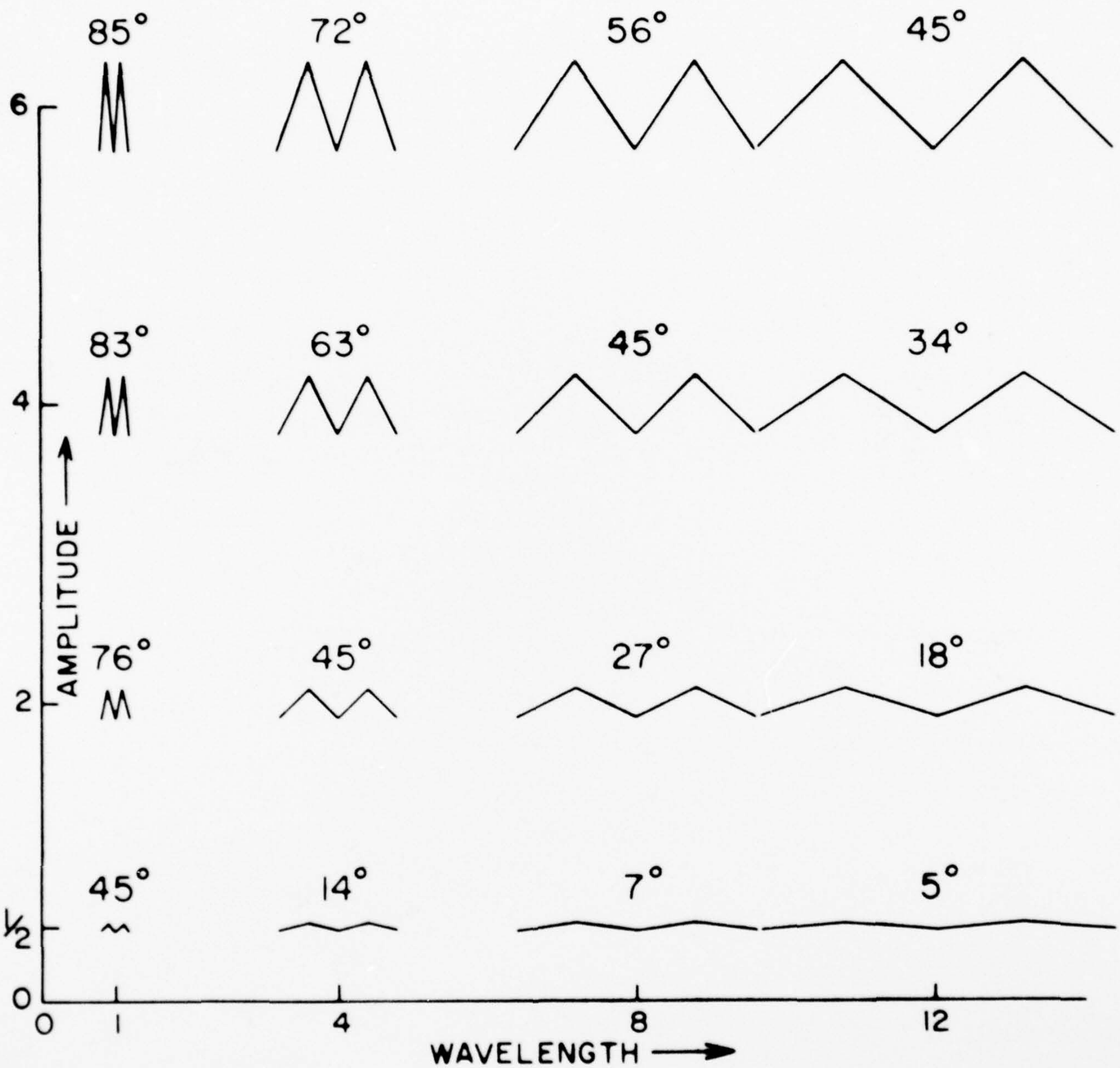


Figure 43 Two-parameter model profiles .

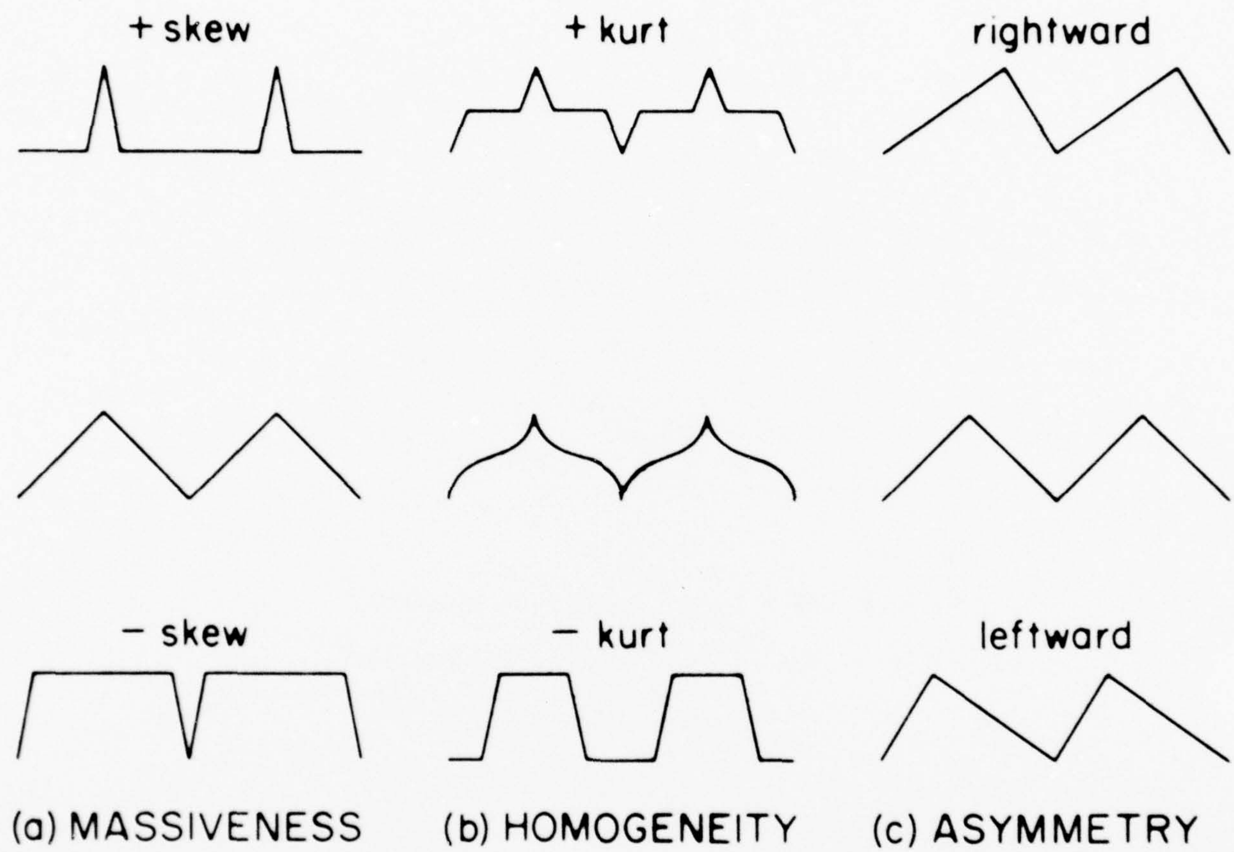
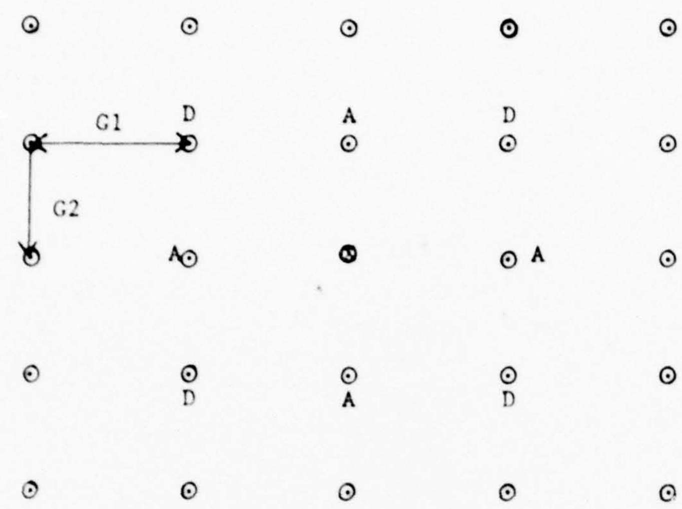


Figure 44 Further variations in profiles.

Figure 45. An altitude matrix



X The current central point

A Axial neighbours of X

D Diagonal " " "

G1, G2 Grid meshes in two orthogonal directions

DAD - AXA - DAD A three x three point submatrix, used in fitting the trend surface to calculate surface derivatives at X.

GLOSSARY

n.b. Only selected terms, relevant to the basic theme of this report, are included. Terms used in relation to other approaches (chapter six) are elucidated in the references given. * See Fig. 45 .

ALTITUDE. Height above mean sea level, measured here in metres (m).

*ALTITUDE MATRIX. A series of altitude values arranged on a rectangular grid, forming one type of DGM. All matrices discussed in this report have a square mesh, i.e. horizontal spacing of values along both axes is equal.

APPARENT GRADIENT. The rate of change of altitude along an arbitrary profile which, in general, is not a slope line. This may be important in communications and trafficability, but not in geomorphology.

ASPECT. The horizontal direction of movement down a slope line. The compass direction (measured in degrees) in which the plane tangent to the surface at a point faces, measured outward from the surface at right angles to the contours.

*AXIAL NEIGHBOURS. The four points nearest a given point in a matrix, along the grid axes.

*CENTRAL POINT. The point for which derivatives are being calculated, central to the 3 x 3 submatrix used.

CONTOUR. A line of equal altitude.

CONVEXITY (CURVATURE). The averaged rate of change of apparent gradient. Height relative to surrounding points on the surface: see also plan c. and profile c.

COURSE LINE. A series of valleys, joined across saddles.

*DIAGONAL NEIGHBOURS. The four points nearest a given point in a matrix, excluding those along the grid axes. In a square mesh these are 1.414 times as distant as the axial neighbours, and are at 45° to the grid axes.

DIGITISATION. Conversion of positional information from analogue into digital form.

DGM (Digital Ground Model : also DTM, Digital Terrain model). A computer-readable representation of the ground surface, from which it is possible to estimate the altitude at any position. A DGM is complete only when accompanied by software to accomplish this interpolation: this is simplest for a grid and most complex for digital contours.

ELEVATION. A term sometimes used synonymously with altitude, but avoided here because of ambiguity between this sense and 'the process of being elevated, i.e. uplifted'.

FREQUENCY DISTRIBUTION. A series of counts of the number of occurrences in a series of adjacent classes of magnitude. These classes should be equal, on the magnitude scale used.

GRADIENT. The rate of change of altitude along a slope line. Expression on an angular rather than a tangent scale is preferred here.

HEIGHT. Vertical separation from some base level, e.g. a river, slope base or (sometimes) sea level, or from the base of the object in question.

HISTOGRAM. The graphic expression of a frequency distribution by a series of parallel bars, with a common baseline. Each bar is proportional in area to the corresponding class count. If the classes and therefore the bars are equal in width, the height of each bar is proportional to the corresponding class count.

KURTOSIS. Crude kurtosis is the fourth moment about the mean of a frequency distribution, standardised by the fourth power of standard deviation to give a dimensionless positive number. Three is subtracted to give kurtosis, which is zero for a Gaussian (normal) distribution. This measures the degree of concentration around the mean, i.e. the extent of the tails (extreme values) relative to the central part of a frequency distribution : high values indicate long tails, usually accompanied by a sharp peak, and negative values indicate tails shorter or thinner than those of a Gaussian distribution with the same standard deviation. A triangular distribution, despite its sharp peak, has negative kurtosis because of its lack of extreme values.

*(GRID) MESH. The horizontal distance to an axial neighbour in an altitude matrix.

kth MOMENT. The mean value of kth powers of values, usually deviations from the mean of a frequency distribution.

ORIENTATION. The compass alignment of lineation, e.g. north-west-southeast (which is identical to southeast-northwest).

PALE. The highest point on a course line joining two pits, a singular point where a contour self-intersects.

PASS. The lowest point on a ridge joining two summits, a singular point where a contour self-intersects.

PIT. A local minimum, below all immediately surrounding altitudes, i.e. surrounded by a higher closed contour .

PLAIN. An area of equal altitude and zero gradient, more extensive than a single data point.

PLAN CONVEXITY. The rate of change of aspect along a contour, in degrees per 100m; negative for concavities.

PROFILE CONVEXITY. The rate of change of gradient along a slope line, in degrees per 100m; negative for concavities.

QUADRATIC. A power series polynomial of second order, i.e. with a squared term.

RESOLUTION. Precision to which measurement is attempted, e.g. nearest degree or fifth-degree. In a DGM, vertical resolution is the smallest recordable altitude difference, and horizontal resolution is the grid mesh.

RELATIONSHIP (between variables). Any connection or covariation which affects the prediction of one variable from another, either in its expected value (e.g. gradient first increases with altitude, then declines) or in its conditional frequency distribution (e.g. profile convexity is less extreme on high gradients).

RIDGE. A linear feature which is a local maximum in one direction, but not in another.

SADDLE. Any pass or pale, a singular point which is a minimum in one direction (the ridge line) and a maximum in another (the course line).

SCALE. The extent of a phenomenon, in linear, areal or volumetric terms. The linear scale of a map is the ratio between a distance on the map and the corresponding distance in the real world, along e.g. the standard parallels or meridians.

SCATTER PLOT. (Scatter diagram). A cartesian plot of the magnitude of one variable against the magnitude of a second variable. Each individual for which the paired values are plotted is represented by a point, and the plot provies a graphic portrayal of the relation between the two variables. If, within the resolution of plotting, several points coincide, further symbols representing 2,3 ... points are required.

SINGULAR POINT. Any point on a surface for which the gradient is momentarily zero: a summit, pit, or saddle, but not a ridge, valley or plain.

SKEWNESS. The third moment of a frequency distribution, standardised by the third power of standard deviation to provide an unbounded dimensionless number. Skewness is zero for any symmetrical distribution, positive where the tail of high values outweighs that of low values, and negative where the tail of low values outweighs that of high values.

SLOPE. Properties of the plane tangent to a point on a surface. These can be specified by a vector in terms of gradient and aspect.

SLOPE LINE. A line of locally greatest rate of change of altitude, along which a frictionless ball starting from rest would roll.

SUMMIT. A local maximum, above all immediately neighbouring altitudes, i.e. surrounded by a lower closed contour.

VALLEY. A linear feature which is a local maximum in one direction, but not in another, and not crossing saddles.

APPENDIX :

Contents of previous reports

STATISTICAL CHARACTERIZATION OF ALTITUDE MATRICES BY COMPUTER

Report 1 1973 Ian S. Evans & Iain Bain 2 tables, 16pp

Page 1 Introduction
1 Investigations of spectral analysis program
6 Binomial smoothing
7 Field studies
8 Further plans
9 Acknowledgements
9 References (12)
10-16 Figures (4)

Report 2 1974 Ian S. Evans & Iain Bain 9 tables, 34pp

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1 Calibration tests on the Rayner-McCalden program for 2-D spectral analysis
Page 2 Tapering (edge smoothing)
2 Addition of zeros
3 Wavelength
6 Orientation
6 Superimposed waves
7 Conclusions
8 Figures (4)
16 Re-processing of altitude matrices
22 A summary of the value of 2-D spectra in general geomorphometry
24 Preliminary calculations of vertical derivatives
27 Relation of field information to the Torridon altitude matrix
Page 27 Matrix overlaying
27 Generation of a terrain matrix
28 Terrain types
31 Matrix overlaying
34 References (10)

Report 3 for 1975 Ian S. Evans 3 tables 24 pp

The effect of resolution on gradients calculated from an altitude matrix

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1 Aim
2 Data
2 Technique
3 Means for individual thinned matrices
4 Means for all possible thinned matrices
6 Standard Deviations
7 Conclusions

AD-A076 558 DURHAM UNIV (ENGLAND) DEPT OF GEOGRAPHY F/G 8/2
STATISTICAL CHARACTERIZATION OF ALTITUDE MATRICES BY COMPUTER. --ETC(U)
1979 I S EVANS DA-ERO-591-73-G0040

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3 OF 3
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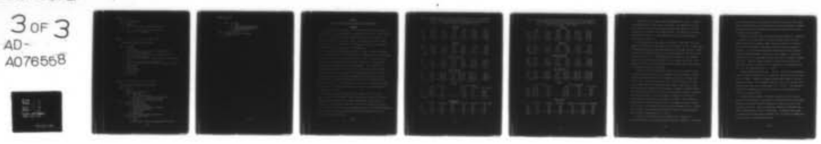

END
DATE
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12-79
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Report 3 (contd)

Page 7 Acknowledgements
8 Tables (3)
11 Figures (6)
17 APPENDIX (relating to the theme of Report 2) :
(A1-A6)
Can we circumvent the stationarity problem?
23-24 References (17) to Appendix

Report 4 1977 Ian S. Evans 14 tables, 56pp

Frequency distributions of gradient

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3 Previous work
6 Methodology for the assessment of transformations
7 Data (i) altitude matrices
10 Data (ii) gradients for variable triangles in a mesh of surface-specific points
12 Data (iii) Relief-based 1km average gradient for Bohemia and Moravia
12 Data (iv) Slope profiles, field-surveyed
15 Effect of horizontal matrix resolution on shape of frequency distributions
15 Conclusions
16 Acknowledgements
17 References (31)
19 Tables (14)
33-56 Figures (24)

Report 5 1978 Margaret Young 9 tables, 27pp

Terrain analysis : program documentation

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1 Mathematical derivations
Page 1 Input data
1 Least squares equation
3 Derivation of descriptive characteristics
5 Conditions for zero gradient points
5 Description of zero gradient points
Page 8 Program descriptions
Page 8 The main program
8 The mapping programs
9 Multiple regressions and histograms
9 Slope plotting program
9 Class limits
10 Organisation
11 Program details and running instructions
Page 11 Main program OMY8
11 Input
12 Output
13 Line printer density map programs OYSM and OYSMA

Page 13 OYSM
14 OYSMA
14 Slope plotting program OYPL
15 Class limits program OMYL
16 Masterfile GEOD
Page 16 References (3)
17 Examples of input and output
19-27 Figures (10)

ADDENDUM

Replicability and scale effects within a large matrix

(Quillan)

Some results are now available for a further data set, kindly provided by Robert Orr of Gestalt International Ltd., Vancouver, Canada. This altitude matrix was created as a by-product of orthophotography for the Quillan area, in the Aude valley southeast of Toulouse, France. It was provided on magnetic tape, as a series of 24 x 24 submatrices. The photoscale spacing of 0.1816 mm gave a ground spacing of 4.544m, equal in both directions.

Since the initial submatrices are rather too small to provide statistics for present purposes, they were reformatted into a single matrix with 552 columns x 1152 rows. This covered an area of 2.508 x 5.235 km. The time required to process such a large matrix with the present program would be excessive, so several parts of the matrix were analysed separately. This revealed that though adjacent submatrices are reconciled to provide first-order continuity, second-order continuity is not present. Steeper gradients occur at almost every 24th column, and much greater convexity and concavity occur every 24th column and row. Inspection of listings of the altitude data showed that the extra differences in height involved were quite small : the discontinuity effect is picked up in the derivatives because the vertical and horizontal resolution of these data is so fine.

The main interest of having such a large data set is that it can be sampled in many different ways. Firstly, this was done to test replicability of the proposed statistics, over a larger data set than those discussed in section (3c). Sampling every fifth row and every fifth column gives a matrix of 22.72 m mesh, with 4 or 5 rows and columns from each of the original submatrices; this is sufficient to make any discontinuity effect negligible, and none is detectable in the maps produced.

Table 14 Summary statistics and linear correlations for four 110 x 229 samples of every fifth point in the Quillan matrix, giving a grid mesh of 22.72m. The displacements are 9.09m : starting points are given on the left (c = column, r = row). Supplied by M. Young.

ALTITUDE

	MEAN	SD	SKEW	KURT	MAX	MIN
c3r3	351.953	74.143	1.080	0.735	629.809	250.520
c5r3	351.735	74.020	1.085	0.752	630.334	250.653
c1r5	352.261	74.124	1.074	0.718	631.585	250.575
c3r5	352.025	74.142	1.079	0.733	631.333	250.688
RANGE	0.526	0.123	0.011	0.034	1.776	0.168

GRADIENT

	MEAN	SD	SKEW	KURT	MAX	MIN
c3r3	13.651	8.596	0.562	-0.401	42.409	0.044
c5r3	13.661	8.596	0.562	-0.406	42.741	0.082
c1r5	13.689	8.598	0.555	-0.420	42.369	0.068
c3r5	13.664	8.593	0.563	-0.400	42.339	0.020
RANGE	0.038	0.005	0.008	0.020	0.402	0.062

PROFILE CONVEXITY

	MEAN	SD	SKEW	KURT	MAX	MIN
c3r3	-2.418	27.564	0.395	5.447	271.057	-174.627
c5r3	-2.412	27.733	0.274	4.649	255.129	-184.323
c1r5	-2.393	27.476	0.340	4.423	248.564	-189.674
c3r5	-2.420	27.474	0.347	4.819	269.008	-165.842
RANGE	0.027	0.259	0.121	1.024	22.493	23.832

PLAN CONVEXITY

	MEAN	SD	SKEW	KURT	MAX	MIN
c3r3	1.166	424.102	7.722	1122.784	26891.625	<-9,999
c5r3	0.482	418.462	11.765	821.983	22957.219	<-9,999
c1r5	1.619	343.578	2.663	234.123	13064.504	-9624.059
c3r5	-1.037	412.397	-22.623	1863.713	13098.934	<-9,999
RANGE	2.656	80.524	34.388	1629.590	13827.121	?

ASPECT

	Unit vectors		Gradient-weighted		
	MEAN	STRENGTH	MEAN	STRENGTH	NO.OF POINTS
c3r3	069.226	.141	075.747	.026	24,516
c5r3	069.121	.137	075.620	.026	24,516
c1r5	068.266	.137	075.220	.026	24,743
c3r5	068.714	.141	075.284	.027	24,516
RANGE	0.960	.004	0.527	.001	227

CORRELATIONS

	ALT:GRAD	ALT:PROFC	ALT:PLANC	GRAD:PROFC	GRAD:PLANC	PROFC:PLANC
c3r3	.410	.167	.044	.021	.026	.111
c5r3	.409	.163	.046	.023	.029	.111
c1r5	.410	.168	.053	.025	.029	.131
c3r5	.410	.168	.046	.025	.035	.117
RANGE	.001	.005	.009	.004	.009	.020

Table 15 Summary statistics and linear correlations for four 69 x 144 samples of every eighth point in the Quillan matrix, giving a grid mesh of 36.35m. The displacements are 9.09m:starting points are given on the left (c = column, r = row).Supplied by M. Young.

ALTITUDE

	MEAN	SD	SKEW	KURT	MAX	MIN
c6r2	351.465	73.949	1.090	0.766	629.149	250.583
c4r4	351.791	74.080	1.084	0.748	629.701	250.639
c2r6	352.167	74.221	1.078	0.728	627.615	250.042
c8r8	351.315	73.801	1.092	0.774	626.314	250.226
RANGE	0.852	0.420	0.014	0.046	3.387	0.397

GRADIENT

	MEAN	SD	SKEW	KURT	MAX	MIN
c6r2	12.930	8.027	0.556	-0.302	39.229	0.117
c4r4	12.941	8.031	0.557	-0.306	39.166	0.055
c2r6	12.934	8.055	0.556	-0.309	39.051	0.127
c8r8	12.977	8.060	0.545	-0.325	39.750	0.053
RANGE	0.047	0.033	0.012	0.023	0.699	0.074

PROFILE CONVEXITY

	MEAN	SD	SKEW	KURT	MAX	MIN
c6r2	-2.264	18.379	0.610	4.932	143.575	-83.271
c4r4	-2.342	18.250	0.516	4.050	122.019	-80.185
c2r6	-2.239	18.261	0.482	4.023	116.774	-88.823
c8r8	-2.287	18.803	0.650	5.341	153.035	-91.808
RANGE	0.103	0.553	0.168	1.318	36.261	11.623

PLAN CONVEXITY

	MEAN	SD	SKEW	KURT	MAX	MIN
c6r2	-2.170	210.664	-5.351	164.641	3256.766	-6401.555
c4r4	3.887	218.845	6.468	236.598	7396.871	-4100.758
c2r6	1.172	202.352	2.232	175.555	6429.672	-5034.195
c8r8	1.238	234.807	7.103	403.013	9783.199	-4444.234
RANGE	6.057	32.455	12.454	238.372	6526.433	2300.797

ASPECT

	Unit vectors		Gradient-weighted		
	MEAN	STRENGTH	MEAN	STRENGTH	NO.OF POINTS
c6r2	067.588	.149	074.995	.027	9514
c4r4	067.556	.151	074.673	.027	9514
c2r6	066.961	.154	074.386	.028	9514
c8r8	065.167	.149	073.332	.026	9514
RANGE	002.421	.005	001.663	.002	0

CORRELATIONS

	ALT:GRAD	ALT:PROFC	ALT:PLANC	GRAD:PROFC	GRAD:PLANC	PROFC:PLANC
c6r2	.429	.223	.095	.027	.076	.139
c4r4	.430	.225	.074	.032	.030	.145
c2r6	.431	.226	.086	.028	.050	.169
c8r8	.431	.223	.084	.031	.045	.133
RANGE	.002	.003	.021	.005	.046	.036

Replicability in the face of small displacements was tested by drawing four separate samples, each of every fifth point. The starting points (and hence the grid incidence) were chosen to keep the four samples at least two columns or two rows apart in the original matrix : the results for these four runs are given in Table 14. Four samples each of every eighth point were drawn similarly (Table 15).

Some characteristics are common to both Tables, being insensitive to the change in resolution from a grid mesh of 22.72m to one of 36.35m. In particular, the altitude distributions are indistinguishable, with ranges from 250 to 630m, a mean of 352m, standard deviation 74m, positive skewness of 1.08, and positive kurtosis of 0.72 to 0.77. The histogram is in fact bimodal, with a primary peak for the valley floor and a smaller one for a plateau around 450m. The disparity in size of the peaks, together with the general skew causes kurtosis to be positive, masking the bimodality.

Gradient is also positively skewed, about +0.56 in both cases, but mean is slightly reduced, from 13.7, to 12.9 degrees for the coarsermesh. Standard deviation is reduced from 8.6 to 8.0 degrees, and kurtosis is weakened from -0.4 to -0.3 : this is a true reflection of a broad distribution with short tails. Maximum gradient is reduced by three degrees. Mean aspect is moved one or two degrees northward and the strength of asymmetry is slightly reduced for unit vectors, while the much lower gradient-weighted strength is unaffected. The standard deviation of profile convexity is reduced to two thirds its 22.72m-mesh value, and that of plan convexity is halved. This confirms the greater effect of mesh on high derivatives. Correlations are rather stronger at 36.35m mesh : this affects the second and third strongest (profile convexity with altitude and with plan convexity) much more than the strongest(gradient with altitude, increased from +.41 to +.43).

In terms of replicability over four distinct grid incidences (i.e. repeated sampling), the results from each grid mesh are comparable. In relative

terms standard deviation is usually most replicable, followed by mean, while higher derivatives give more variable results, kurtosis being worst. Maximum and minimum vary from as replicable as the mean for altitude, to as bad as kurtosis for the convexities.

However, since some values are near-zero and can be positive or negative, it is better to view replicability in terms of actual values, or of the possible range of the statistic, rather than as a coefficient of variation. In this case, the maximum is always more variable than mean or standard deviation. All the altitude and gradient statistics are highly replicable (mean and standard deviation to within 1m or 0.05 degress : skewness and kurtosis within 0.05): mean aspect is replicable within 2.4 degrees, strength within .005. Mean and standard deviation of profile convexity are quite replicable - within a few percent or 0.6 degrees/100m - but skewness varies up to 0.17 (30%) and kurtosis up to 1.3 (27%).

The main problem comes with plan convexity. Here there is one aberrant value of standard deviation for 22.72 m mesh, but variation is over 10% also for 36.35 m mesh. Variation in skewness, kurtosis and extrema is such as to render them useless. The variations observed in section (3c) were thus due to extreme values rather than to small sample size. Clearly, it is essential to transform the measurement scale of plan convexity so that statistics can cope with the extreme values.

Except for plan convexity, all variations are worse for the coarser matrix, but the difference is not drastic. Correlations are quite replicable, especially that between altitude and gradient (±.002) which is by far the strongest correlation. Those involving plan convexity are predictably the least stable, especially with the coarser mesh. In general, these results are reassuring and support those of section (3c). The effect of varying grid mesh will next be investigated more fully.